This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSAC.2016.2611847, IEEE Journal on Selected Areas in Communications

Energy Efficient Joint Scheduling and Power Control in Multi-Cell Wireless Networks

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I. Abstract

Traditional design of wireless networks mainly focuses on system capacity and spectral efficiency. As green networking is an inevitable trend, energy-efficient design for future wireless networks becomes paramount. In this paper, we address energy efficient resource management in downlink Orthogonal Frequency Division Multiple Access (OFDMA) networks. The focus is targeted towards multi-cell networks, which are composed of multiple Base Stations (BSs) sharing the available radio resources. Consequently, greater emphasis is given to techniques that take inter-cell interference into account. Resource management in our context refers to the task of allocating the radio resources in order to maximize energy efficiency. We devise resource management techniques that jointly tackle the problems of scheduling and power control. Accordingly, we adopt two different approaches: a *centralized approach* where BSs coordinate in order to reach a globally optimal energy efficient solution; and a *distributed approach* where BSs selfishly strive to maximize their own energy efficiency. We portray the centralized approach as a convex optimization problem; whereas, we have recourse to non-cooperative game theory to model the distributed approach. Particularly, we show that the non-cooperative game converges to a unique Nash equilibrium in low and high interference scenarios. We perform thorough numerical simulations to quantify the discrepancy between the centralized and distributed approaches, and identify the conditions where they have precedence over the state-of-the-art. Moreover, the simulation results highlight the fast convergence of our algorithms, which is a precious asset for realistic deployments.

***Index Terms*—Convex optimization, energy efficiency, multi-cell OFDMA, Nash equilibrium, non-cooperative game theory, power control, scheduling, wireless networks.**

II. Introduction

The escalation of power consumption in wireless networks has led to a great amount of greenhouse gas emission and a large operational expenditure for network operators. Increasing attention has been paid to green networking and has prompted new waves of research and standard development activities [1]–[3]. In particular, it is reported that over 80% of power consumption in mobile telecommunications is squandered in the radio access network, more specifically at the Base Station (BS) level [4]. Consequently, acute focus is devoted to energy efficient radio resource management to keep pace with the pervasive green trend in networking. In this context, joint power control and scheduling is one of the most promising approaches to increase energy efficiency [5]–[8].

Orthogonal Frequency Division Multiple Access (OFDMA) is widely accepted as the access scheme for many wireless systems, such as 4G downlink systems, thanks to its numerous merits, especially in terms of spectral efficiency. In such systems, the intra-cell interference is mostly mitigated and can be ignored. However, the inter-cell interference is a momentous problem in OFDMA-based networks, owing to the expected densification of BSs and the support of universal frequency reuse in order to increase spectral efficiency [9]. As inter-cell interference cannot be canceled by signal processing techniques, intelligent Radio Resource Management (RRM) is required. The latter should lessen the impact of inter-cell interference and increase energy efficiency. Therefore, particular attention must be dedicated to energy-efficient RRM in OFDMA-based multi-cell networks. A successful approach to this challenging design must exhibit satisfying performances in terms of energy efficiency, computation complexity, and convergence time.

In general, existing RRM techniques are divided into two broad categories: *centralized approaches* and *distributed approaches* [10]. Centralized approaches enable to compute a global energy efficient state of the network. Such solutions necessitate coordination and signaling between the BSs. Distributed approaches enable autonomous BSs to maximize their own energy efficiency leading to complexity reduction at the cost of lower efficiency.

In this paper, we tackle the energy efficient joint scheduling and power control problem in multi-cell OFDMA networks. We formulate the centralized approach as a convex optimization problem and devise algorithms that enable to rapidly compute an optimal solution. We also propose a distributed approach for this problem in which we have recourse to non-cooperative game theory. In particular, we introduce distributed algorithms that provably converge to a Nash Equilibrium (NE). We perform thorough numerical simulations for assessing the performance of our approaches in various interference conditions.

The rest of the paper is organized as follows. In Section III, related work is explored. In Section IV, we put forward the models adopted for the network power consumption, the Signal-to-Interference-and-Noise-Ratio (SINR), and the net-

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This work is partially supported by NSFC (Grant No. 61471089).

Manuscript received February 1, 2016; revised May 21, 2016.

work utility function. In Section V and Section VI, we detail the centralized approach and the projected-gradient based solution, respectively. The distributed approach is introduced in Section VII and the corresponding algorithmic solution as well as the convergence proofs are explained in Section VIII. In Section IX, we present extensive numerical results that corroborate the relevance of our devised approaches. Finally, we outline practical guidelines to implement the proposed algorithms in Section X and conclude in Section XI.

III. Related Work

The inevitable need to improve energy efficiency in cellular networks has been established, and has led to numerous research works on the hot topic of green radio communications. Energy efficient RRM techniques are covered in [10] where a pertinent taxonomy of power saving strategies in wireless networks is introduced. The recent findings in the area of multi-cell networks are surveyed in [11]. Further, green scheduling and power control for both classic and heterogeneous cellular networks are covered in [12].

Maximizing the energy efficiency in a wireless network is equivalent to maximizing the amount of data bits successfully delivered to the receiver for each consumed power unit. Given the non-convex, fractional nature of this objective, an effective mathematical tool for energy efficiency maximization is fractional programming in optimization theory. The latter provides algorithms with polynomial complexity to maximize fractional functions with a concave numerator and a convex denominator [13]. However, even this robust tool fails when multi-cell networks are considered, as interference makes the numerator of energy efficiency non-concave. Consequently, the majority of related work considers the single cell scenario where inter-cell interference is inadequately neglected [14]–[18]. In the following, we discuss existing approaches in the field of energy efficient radio resource allocation, then highlight the major contributions of our work.

A. Centralized Approaches

1) The single-cell case: Starting from the seminal work in [14], the energy efficiency problem is proved to be quasi-concave. Thus, a unique global optimal power allocation always exists. Based on these findings, a plethora of work has been developed as in [15]–[18]. In [15], the problem is modeled as the maximization of energy efficiency under QoS requirements: the data rate per user must be greater than a given threshold. Both scheduling and power control are studied on the downlink of a single cell OFDMA network. The optimal solution is first given, then a low-complexity suboptimal solution is developed by solving iteratively the two problems at hand. In [16], a waterfilling-like solution is proposed to the power allocation problem. Further, the work in [17] integrates simultaneously spectral and energy efficiency in the resource allocation objective. This enables to exploit the latter tradeoff by balancing power consumption and occupied bandwidth. In [18], proportional rate constraints are added and a suboptimal two-step approach is adopted to solve the scheduling and the power allocation problems.

The studies on energy efficiency have also typically dealt with fairness between users. Since the efficiency of different users cannot be maximized simultaneously, the work in [19] investigates the use of individual energy efficiency rather than overall efficiency. Similarly, the authors in [20] replace the network energy efficiency by a max-min objective.

Recently, holistic approaches for energy efficiency have been introduced. In [21], the authors take into account the energy on the transmitter and the receiver side. Once again, for tractability, the problem is suboptimally solved in two steps where scheduling and power allocation are iteratively operated. Both transmission sides are also considered in [22], focusing on joint uplink and downlink resource allocation for energy efficient carrier aggregation.

2) The multi-cell case: Energy efficiency have recently been studied in the context of multi-cell wireless networks. In [23], [24], the downlink of a multi-cell OFDMA system is considered, wherein a cluster of coordinated BSs perform energy efficient scheduling and power control under a per sub-carrier power constraint. A heuristic method to solve the first-order optimality conditions of this mixed integer continuous problem is provided. However, a solution is only proposed in the noise-limited regime, which boils down to the single cell scenario. In [25], both centralized and distributed approaches for energy efficient power control are explored. Minimum-rate constraints are imposed to ensure fairness in resource sharing. Optimal energy efficiency is reached in the centralized approach with a reasonable complexity. Moreover, convergence to NE is established in the distributed approach. However, this is done solely in the single Resource Block (RB) scenario. For the multiple RB scenario, simplified assumptions are introduced to devise appropriate solutions.

In the state-of-the-art, various ways to achieve energy efficiency are introduced. For instance, energy efficiency can be treated as a multi-objective optimization. In [26], a bi-objective optimization problem is formulated for each BS, which aims at both maximizing the BS throughput contribution and minimizing its total power consumption. The authors transform the bi-objective problem into an equivalent scalar objective problem (via the so-called Pascoletti and Serafini method). In [27], the authors assume that efficiency can be simply achieved by minimizing the power consumption subject to rate constraints. The work in [28] resorts to stochastic geometry to obtain tractable analytical results for energy efficient multi-cell networks. Nonetheless, power is set only in the central BS, which hinders the benefits of multi-cell power control.

B. Distributed Approaches

Distributed scheduling and power allocation for energy efficiency in multi-cell OFDMA networks is the subject of convergent research work [29]–[33]. In these papers, BSs are enabled to determine autonomously the resource allocation based on a local view of the network. The interaction between selfish BSs is typically modeled using non-cooperative game theory, and scientific effort is made to characterize attractive solutions corresponding to NE.

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In this common framework, interference-limited scenarios are considered in [29] and the existence of NE is proved. However, the uniqueness of NE is established only when a single sub-channel is used or when the channel experiences flat fading. Pursuing the previous investigations, the work in [30] sheds light on the dynamics of the spectral and energy efficiency tradeoff. All users are assumed to experience equal interference and channel gains. This simplified model overlooks inter-cell interference despite its severe impact when transmission is operated on multiple sub-channels. In [31], iterative Best Response dynamics are used to reach the NE but without proof of convergence. Further, the series of modifications to the original problem and the decoupling of scheduling and power control make the proposed solution heuristic. In [32], discrete power levels are considered. A suboptimal solution is proposed due to the NP-hardness of the problem where power control and scheduling are again addressed iteratively. The same approach is used in [33] where the authors put forward a potential game for power control.

The context of cooperative networks is explored in [34]. In such networks, transmit data can be aided by several relays. The problem is formulated as a non-cooperative power control game with discrete power levels. A Q-learning algorithm is proposed to converge to the mixed NE of the finite game. In [35], the power control on the uplink of heterogeneous networks is investigated with an innovative approach that seeks Debreu equilibrium.

C. Our Contributions

This work presents a joint approach for scheduling and power control in wireless networks. The general contribution consists in formulating and solving the energy efficient resource management problem in multi-cell OFDMA networks using both centralized and distributed approaches. We summarize in this section the major contributions and the differences from existing work.

- Our formulation is tailored to multi-cell networks, contrary to a large part of research work, where only a single OFDMA cell is considered [36], [15], [21], [22], [23], [24]. Such approaches neglect the impact of inter-cell interference, which appears as a major shortcoming when allocating resources in a network with dense spectrum reuse. To highlight the latter, we compare our algorithms with the state-of-the-art single cell approaches. The results show the relevance of our realistic contributions especially in high interference scenarios.
- In our work, we tackle the joint problem of scheduling and power control in OFDMA-based wireless networks. Our formulation enables to mathematically separate the two problems without loosing optimality. To our knowledge, the state-of-art work addresses scheduling and power control iteratively leading to suboptimal results [15], [18], [21], [23], [24], [31], [32].
- Our salient objective is to address the radio resource allocation according to the two widely adopted points of view in wireless networks: the centralized approach where a global energy efficient scheduling and power control is computed on a central controller. The distributed approach, modeled as a non-cooperative game, where autonomous BSs selfishly maximize their energy efficiency. Moreover, we provide numerical results that assess the difference between the two approaches.
- We perform mathematical transformations on the centralized and distributed formulations in order to obtain convex optimization problems. Particularly, we make use of the Dinkelbach [13] method followed by a geometric variable change. These transformations enable us to devise efficient algorithmic solutions, based on the projected subgradient [37].
- We prove mathematically that the centralized and distributed approaches converge to a global optimum, or to pure Nash equilibrium, respectively. In particular, we prove that the non-cooperative distributed game is supermodular. Such games are known to converge to pure Nash equilibrium by applying simple Best Response dynamics. More importantly, we provide a formal proof of the uniqueness of the equilibrium in low and high interference scenarios. Using simulations means, we show the rapid convergence of both approaches, which is a precious asset for realistic deployments.

The work in [25] is the closest to our paper where both centralized and distributed approaches for energy efficient power control are explored. However, the study does not encompass the scheduling problem. To ensure fairness, minimum-rate constraints are imposed, whereas we chose to apply proportional fairness to realize equity in resource sharing. More importantly, simplifying assumptions are introduced in [25] to circumvent the analytical complexity of the multiple RBs scenario. On the contrary, our work devises optimal algorithmic solutions for the general problem that are valid in the realistic multiple RB setting of an OFDMA network.

IV. The Network Model

We consider a cellular network comprising a set of BSs denoted by $\mathcal{J}$. We focus on the downlink in this paper where OFDMA is used as the multiple access scheme. The time and frequency radio resources are grouped into time-frequency RBs, as for instance in LTE networks. An RB is the smallest radio resource unit that can be scheduled to a mobile user. Each RB consists of N_s OFDMA symbols in the time dimension and N_f sub-carriers in the frequency dimension (in LTE, $N_s = 7$ as in the most used formats and $N_f = 12$). The set of RBs is denoted by $\mathcal{K}$, and the set of users is denoted by $\mathcal{I}$. The notation used in this document is depicted in Table I. In the sequel, we make the following assumptions:

1) we consider a *fixed user to BS association* and denote by $\mathcal{I}(j)$ the set of users associated with BS $j \in \mathcal{J}$. Each user typically compares the received signal power from each BS and chooses to connect with the best received BS;
2) we consider *saturation mode for downlink traffic* where each BS has persistent traffic towards its users. We also assume that all RBs are assigned on the downlink at each scheduling interval.

TABLE I
SETS, PARAMETERS AND VARIABLES NOTATION

$\mathcal{J}$	Set of BSes.
$\mathcal{I}$	Total set of users.
$\mathcal{I}(j)$	Set of users associated to BS j.
$\mathcal{K}$	Set of RBs.
$\|.\|$	Number of elements in a set.
π_{jk}	Transmit power of BS j on RB k.
θ_{ik}	Percentage of time user i is associated with RB k.
p^{min}	Minimum power on each RB.
p_j^{max}	Maximum power of BS j.

A. The Power Consumption Model

The transmit power of each BS is given to the resource blocks serving the users in the cell. Assuming the utilization of a power allocation algorithm, the total transmit power of BS j is the sum of the transmit power on each of the RBs $k \in \mathcal{K}$, written as $\sum_{k\in\mathcal{K}} \pi_{jk}$, where π_{jk} denotes the transmit power of BS j on RB k. The power consumption of BS $j \in \mathcal{J}$ is modeled as a linear function of the transmit power as below [38]:

$$p_j = p_j^1(\sum_{k\in\mathcal{K}} \pi_{jk}) + p_j^0. \tag{1}$$

The coefficient p_j^1 accounts for the power consumption that scales with the transmit power due to radio frequency amplifier and feeder losses while p_j^0 models the power consumed independently of the transmit power due to signal processing and site cooling.

In the following, we consider that the power on each RB is larger than a predefined value denoted p^{min}, and the transmit power of BS j is lower than p_j^{max}. In practice, these bounds are related to hardware limitations. Further, this leads to a more balanced power allocation on the different subcarriers, allowing a simpler design of the transmit amplifiers. The total power consumed in the network is given by the sum of the power in each BS:

$$P = \sum_{j\in\mathcal{J}} p_j = \sum_{j\in\mathcal{J},k\in\mathcal{K}} p_j^1 \pi_{jk} + \sum_{j\in\mathcal{J}} p_j^0. \tag{2}$$

B. The SINR model

Given a user i associated with BS j (*i.e.*, $i \in \mathcal{I}(j)$), the SINR of this user when served on RB k is given by:

$$\rho_{ijk} = \frac{\pi_{jk} G_{ijk}}{N_0 + \sum_{j'\neq j} \pi_{j'k} G_{ij'k}}, \tag{3}$$

where G_{ijk} is the channel power gain of user i on RB k of BS j (*e.g.*, computed as an average power gain over the subcarriers in the RB), and N_0 is the noise power. The latter is assumed, without loss of generality, to be the same for the all users on all RBs. In our work, we rely on perfect channel state information to infer the SINR. Authors in [39] provide models to account for imperfect channel state and study the impact on energy efficiency.

C. The Network Utility model

Let θ_{ik} denote the percentage of time user i is scheduled on RB k. The utility function of the network is defined by the following:

$$\begin{aligned} R &= \sum_{j\in\mathcal{J},i\in\mathcal{I}(j)} \sum_{k\in\mathcal{K}} \log(\theta_{ik}\rho_{ijk}) \\ &= \sum_{j\in\mathcal{J},i\in\mathcal{I}(j),k\in\mathcal{K}} \log(\frac{\theta_{ik}\pi_{jk}G_{ijk}}{N_0 + \sum_{j'\neq j}\pi_{j'k}G_{ij'k}}), \end{aligned} \tag{4}$$

with $0 \leq \theta_{ik} \leq 1$. Note that R also corresponds to the logarithm of the geometric mean of SINR:

$$R = \log(\prod_{j\in\mathcal{J},i\in\mathcal{I}(j),k\in\mathcal{K}} \theta_{ik}\rho_{ijk}) \tag{5}$$

As introduced in [40], the logarithmic utility function is intimately associated with the concept of proportional fairness. Thus, the expression of R yields a proportional fair rate service by each BS for users on each RB. This network utility formulation is technology-agnostic. In the context of LTE networks, the mapping between the SINR and the throughput of each user can be derived according to the appropriate modulation and coding scheme. Inevitably, improving this network utility boils down to improving the user throughput.

V. CENTRALIZED FORMULATION OF THE ENERGY EFFICIENCY PROBLEM

In this section, we introduce a centralized formulation of the joint scheduling and power allocation problem in a multi-cell network. The objective is to maximize the energy efficiency expressed as the network utility per power unit. Accordingly, we maximize the ratio of the network utility as given in (4) and the total power consumption in all BSs. Our energy efficiency objective represents a benefit-cost ratio as established in [39]. The benefit measures the income of the system as the amount of data that can be transmitted to the receiver in a time interval while ensuring proportional fairness. The cost is represented by the total amount of energy consumed to run the system. The centralized optimization problem is given in (6) and denoted by $(\mathcal{P})$ in the sequel.

$$\begin{aligned} (\mathcal{P}): \quad & \max_{\boldsymbol{\theta},\boldsymbol{\pi}} \left\{ \frac{R(\boldsymbol{\theta},\boldsymbol{\pi})}{P(\boldsymbol{\pi})} \right\} = \\ & \max_{\boldsymbol{\theta},\boldsymbol{\pi}} \left\{ \frac{\sum_{j\in\mathcal{J},i\in\mathcal{I}(j),k\in\mathcal{K}} \log(\theta_{ik}\rho_{ijk})}{\sum_{j\in\mathcal{J},k\in\mathcal{K}} p_j^1\pi_{jk} + \sum_{j\in\mathcal{J}} p_j^0} \right\} && \text{(6a)} \\ \text{subject to} \quad & \sum_{k\in\mathcal{K}} \theta_{ik} \leq 1, \ \forall j \in \mathcal{J}, \forall i \in \mathcal{I}(j), && \text{(6b)} \\ & \sum_{k\in\mathcal{K}} \pi_{jk} \leq p_j^{max}, \ \forall j \in \mathcal{J}, && \text{(6c)} \\ & \pi_{jk} \geq p^{min}, \ \forall j \in \mathcal{J}, \forall k \in \mathcal{K}, && \text{(6d)} \\ & 0 \leq \theta_{ik} \leq 1, \ \forall j \in \mathcal{J}, \forall i \in \mathcal{I}(j), \forall k \in \mathcal{K}. && \text{(6e)} \end{aligned}$$

The objective function (6a) consists in maximizing the ratio of the network utility, yielding a proportional fair service, and the total power consumed in the network. The optimization variables are $\pi_{jk}, \forall j \in \mathcal{J}, \forall k \in \mathcal{K}$ and $\theta_{ik}, \forall i \in \mathcal{I}(j), \forall k \in \mathcal{K}$, also represented by a vector notation as $\boldsymbol{\theta}$ and $\boldsymbol{\pi}$. Constraints

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(6b) ensure that a user shares its time on the available RBs. Constraints (6c) ensure that the power consumption of each BS j is lower than p_j^{max}. Constraints (6d) and (6e) define the bounds on the optimization variables π_{jk} and θ_{ik}.

A. Decomposition of the Centralized Energy Efficiency Problem

The problem $(\mathcal{P})$ in (6) cannot be solved in a straightforward manner as it is non-convex. However, as it is a fractional problem, it can be transformed into a parametrized convex programming problem as follows [13]:

$$(\hat{\mathcal{P}}) : F(\eta) = \max_{\boldsymbol{\theta},\boldsymbol{\pi}} \{R(\boldsymbol{\theta},\boldsymbol{\pi}) - \eta P(\boldsymbol{\pi})\} \tag{7a}$$

$$\text{subject to } (6b), (6c), (6d), (6e). \tag{7b}$$

where η is a non-negative parameter.

Let us denote by η^* the maximum energy efficiency of the original problem $(\mathcal{P})$ in (6) which can be expressed as:

$$\eta^* = \frac{R(\boldsymbol{\theta}^*,\boldsymbol{\pi}^*)}{P(\boldsymbol{\pi}^*)} = \max_{\boldsymbol{\theta},\boldsymbol{\pi}} \left\{ \frac{R(\boldsymbol{\theta},\boldsymbol{\pi})}{P(\boldsymbol{\pi})} \right\} \tag{8}$$

$(\hat{\mathcal{P}})$ achieves the maximum energy efficiency η^* if and only if:

$$\begin{aligned} F(\eta^*) &= \max_{\boldsymbol{\theta},\boldsymbol{\pi}} \{R(\boldsymbol{\theta},\boldsymbol{\pi}) - \eta^* P(\boldsymbol{\pi})\} \\ &= R(\boldsymbol{\theta}^*,\boldsymbol{\pi}^*) - \eta^* P(\boldsymbol{\pi}^*) \\ &= 0 \end{aligned} \tag{9}$$

Dinkelbach [13] demonstrated that $F(\eta)$ is continuous and strictly monotonically decreasing in η, thus it has a unique root η^* that can be derived using an iterative algorithm, known as the Dinkelbach method. Moreover, the optimal solution set $\{\boldsymbol{\theta}^*,\boldsymbol{\pi}^*\}$ of $(\mathcal{P})$ is the same as that of $(\hat{\mathcal{P}})$ with $\eta = \eta^*$.

Algorithm 1 Iterative Solution of the Energy Efficiency problem

Require: Maximum tolerance $\epsilon \geq 0$, iteration $n \leftarrow 0$, initial value η^0

1: **repeat**
2: $\eta \leftarrow \eta^n$
3: Solve $(\hat{\mathcal{P}})$ in (7)
4: $(\boldsymbol{\theta}^n,\boldsymbol{\pi}^n) \leftarrow \arg\max_{\boldsymbol{\theta},\boldsymbol{\pi}} \{R(\boldsymbol{\theta},\boldsymbol{\pi}) - \eta P(\boldsymbol{\pi})\}$
5: $F^n \leftarrow \max_{\boldsymbol{\theta},\boldsymbol{\pi}} \{R(\boldsymbol{\theta},\boldsymbol{\pi}) - \eta P(\boldsymbol{\pi})\}$
6: $\eta^{n+1} \leftarrow \frac{R(\boldsymbol{\theta}^n,\boldsymbol{\pi}^n)}{P(\boldsymbol{\pi}^n)}$
7: $n \leftarrow n+1$
8: **until** $F^n \leq \epsilon$

The proposed iterative procedure is summarized in Algorithm (1) and the convergence to the optimal energy efficiency is guaranteed if the inner problem $(\hat{\mathcal{P}})$ is solved to optimality in each iteration in Line 3. The value of η is updated in Line 6 and the convergence is attained for $F^n \leq \epsilon$, with ϵ a given maximum tolerance.

We note that the term $R(\boldsymbol{\theta},\boldsymbol{\pi})$ in the objective function of $(\hat{\mathcal{P}})$ is a sum of functions in $\boldsymbol{\theta}$ and $\boldsymbol{\pi}$ and can be written as:

$$\begin{aligned} R(\boldsymbol{\theta},\boldsymbol{\pi}) &= \sum_{j\in\mathcal{J},i\in\mathcal{I}(j),k\in\mathcal{K}} \log(\theta_{ik}\rho_{ijk}) \\ &= \sum_{j\in\mathcal{J},i\in\mathcal{I}(j),k\in\mathcal{K}} \log(\theta_{ik}) + \log(\rho_{ijk}) \\ &= R_1(\boldsymbol{\theta}) + R_2(\boldsymbol{\pi}) \end{aligned} \tag{10}$$

As the constraints of $(\hat{\mathcal{P}})$ involve either $\boldsymbol{\theta}$ or $\boldsymbol{\pi}$ separately, therefore $(\hat{\mathcal{P}})$ is block separable into two optimization sub-problems $(\hat{\mathcal{P}}_1)$ and $(\hat{\mathcal{P}}_2)$ given in (11) and (12) respectively. Note that this decomposition preserves the optimality of the solution [41].

$$(\hat{\mathcal{P}}_1(\boldsymbol{\theta})) : \quad \max_{\boldsymbol{\theta}} \{ \sum_{j\in\mathcal{J},i\in\mathcal{I}(j),k\in\mathcal{K}} \log(\theta_{ik}) \} \tag{11a}$$

$$\text{subject to} \quad \sum_{k\in\mathcal{K}} \theta_{ik} \leq 1, \ \forall i \in \mathcal{I}(j), \forall j \in \mathcal{J}, \tag{11b}$$

$$0 \leq \theta_{ik} \leq 1, \ \forall j \in \mathcal{J}, \forall i \in \mathcal{I}(j), \forall k \in \mathcal{K}. \tag{11c}$$

$$\begin{aligned} (\hat{\mathcal{P}}_2(\boldsymbol{\pi})) : \quad \max_{\boldsymbol{\pi}} \{ &\sum_{j\in\mathcal{J},i\in\mathcal{I}(j),k\in\mathcal{K}} \log(\rho_{ijk}) \\ &- \eta(\sum_{j\in\mathcal{J},k\in\mathcal{K}} p_j^1 \pi_{jk} + \sum_{j\in\mathcal{J}} p_j^0) \} \end{aligned} \tag{12a}$$

$$\text{subject to} \quad \sum_{k\in\mathcal{K}} \pi_{jk} \leq p_j^{max}, \ \forall j \in \mathcal{J}, \tag{12b}$$

$$\pi_{jk} \geq p^{min}, \ \forall j \in \mathcal{J}, \forall k \in \mathcal{K}. \tag{12c}$$

Similarly, $(\hat{\mathcal{P}}_1)$ can be decomposed into $|J|$ per-cell scheduling problems. Each problem, denoted $(\hat{\mathcal{P}}_{1j})$ and given in (13), computes the percentage of time user i is served on each RB k in BS j. $(\hat{\mathcal{P}}_{1j})$ for each BS j can be solved independently of Algorithm (1). Further, $(\hat{\mathcal{P}}_2)$ is a multi-cell power allocation algorithm and should be solved at each iteration of the Dinkelbach algorithm. Therefore, the following section deals with these two sub-problems and provides algorithmic solutions.

$$(\hat{\mathcal{P}}_{1j}(\boldsymbol{\theta})) : \quad \max_{\boldsymbol{\theta}} \{ \sum_{i\in\mathcal{I}(j),k\in\mathcal{K}} \log(\theta_{ik}) \} \tag{13a}$$

$$\text{subject to} \quad \sum_{k\in\mathcal{K}} \theta_{ik} \leq 1, \ \forall i \in \mathcal{I}(j), \tag{13b}$$

$$0 \leq \theta_{ik} \leq 1, \ \forall i \in \mathcal{I}(j), \forall k \in \mathcal{K}. \tag{13c}$$

VI. Solving the Centralized Energy Efficiency Problem

As introduced in the previous section, the centralized optimization formulation can be decomposed into a per-cell scheduling sub-problem and a multi-cell power control sub-problem. This decomposition preserves the optimality and an optimal solution of each sub-problem leads to a global one. In this section, we introduce algorithmic methods that compute optimal solutions for the scheduling and power control problems, respectively.

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A. Solution of the Per-Cell Scheduling Problem

The per-cell scheduling problem ($\hat{\mathcal{P}}_{1j}$) in (13) is a convex optimization problem: the objective function (13a) is concave (sum of concave functions) and all constraints are linear.

Theorem 6.1: The optimal solution of the per-cell scheduling problem ($\hat{\mathcal{P}}_{1j}$) for each BS j is given by:

$$\forall i \in \mathcal{I}(j), \forall k \in \mathcal{K}, \theta^*_{ik} = \frac{1}{|\mathcal{K}|}. \tag{14}$$

Proof: See Appendix A. □

B. Solution of the Multi-Cell Power Control Problem

The multi-cell power control problem ($\hat{\mathcal{P}}_2$) in (12) is a non-linear, and apparently difficult, non-convex optimization problem. However, it can be transformed into a convex optimization problem in the form of geometric programming; hence it can be very efficiently solved for global optimality even with a large number of users.

1) Convex transformation: In Appendix B, we demonstrate that problem ($\hat{\mathcal{P}}_2$) in (12) can be transformed into a convex optimization via a change of variables $\tilde{\pi}_{jk} = \log(\pi_{jk})$. We obtain the following convex problem ($\hat{\mathcal{P}}_2(\tilde{\boldsymbol{\pi}})$), written with $\tilde{N}_0 = \log(N_0)$ and $\tilde{G}_{ijk} = \log(G_{ijk})$:

$$\begin{aligned}(\hat{\mathcal{P}}_2(\tilde{\boldsymbol{\pi}})):\quad \max_{\tilde{\boldsymbol{\pi}}}\{ &\sum_{j\in\mathcal{J},i\in\mathcal{I}(j),k\in\mathcal{K}} (\tilde{\pi}_{jk} + \tilde{G}_{ij'k}) \\ &- \sum_{j\in\mathcal{J},i\in\mathcal{I}(j),k\in\mathcal{K}} \log(\exp(\tilde{N}_0) \\ &- \eta(\sum_{j\in\mathcal{J},k\in\mathcal{K}} p^1_j \exp(\tilde{\pi}_{jk}) + \sum_{j\in\mathcal{J}} p^0_j)\}\end{aligned} \tag{15a}$$

subject to

$$\log(\sum_{k\in\mathcal{K}} \exp(\tilde{\pi}_{jk})) - \log(p^{max}_j) \le 0, \forall j \in \mathcal{J}, \tag{15b}$$

$$-\tilde{\pi}_{jk} + \log(p^{min}) \le 0,\ \forall j \in \mathcal{J}, \forall k \in \mathcal{K}. \tag{15c}$$

2) Projected subgradient-based algorithm: In order to solve the convex problem ($\hat{\mathcal{P}}_2(\tilde{\boldsymbol{\pi}})$) in a centralized fashion, we can have recourse to the subgradient method for constrained optimization. This algorithm takes the following form:

$$\tilde{\pi}_{jk}(t+1) = \tilde{\pi}_{jk}(t) + \delta(t)\tilde{g}(t),$$

where t is the iteration number, $\delta(t)$ a step size, and $\tilde{g}(t)$ a subgradient of the objective function in (15a). We take:

$$\tilde{g}(t) = \nabla_{\tilde{\pi}_{jk}}(R_2(\tilde{\boldsymbol{\pi}}) - \eta P(\tilde{\boldsymbol{\pi}})) \tag{16}$$

In order to express the subgradient in (16), let us start by computing the derivative of the objective function in (15a) with respect to $\tilde{\pi}_{jk}$:

$$\begin{aligned}&\nabla_{\tilde{\pi}_{jk}}(R_2(\tilde{\boldsymbol{\pi}}) - \eta P(\tilde{\boldsymbol{\pi}})) \\ &= |\mathcal{I}(j)| - \sum_{\substack{l\in\mathcal{J}\\ l\neq j}} \sum_{i\in\mathcal{I}(l)} \frac{G_{ijk}\exp(\tilde{\pi}_{jk})}{\sum_{\substack{j'\in\mathcal{J}\\ j'\neq l}} \exp(\tilde{\pi}_{j'k})G_{ij'k} + N_0} \\ &\quad - \eta p^1_j \exp(\tilde{\pi}_{jk}) \\ &= |\mathcal{I}(j)| - \pi_{jk} \sum_{\substack{l\in\mathcal{J}\\ l\neq j}} \sum_{i\in\mathcal{I}(l)} \frac{G_{ijk}}{\sum_{\substack{j'\in\mathcal{J}\\ j'\neq l}} \pi_{j'k}G_{ij'k} + N_0} - \eta p^1_j \pi_{jk}\end{aligned}$$

Coming back to the solution space in $\boldsymbol{\pi}$ instead of $\tilde{\boldsymbol{\pi}}$, the derivative of the objective function in (12a) with respect to π_{jk} is given by:

$$\begin{aligned}&\nabla_{\pi_{jk}}(R_2(\boldsymbol{\pi}) - \eta P(\boldsymbol{\pi})) \\ &= \nabla_{\pi_{jk}}(\sum_{j\in\mathcal{J},i\in\mathcal{I}(j),k\in\mathcal{K}} \log(\rho_{ijk}) \\ &\quad - \eta(\sum_{j\in\mathcal{J},k\in\mathcal{K}} p^1_j \pi_{jk} + \sum_{j\in\mathcal{J}} p^0_j)) \\ &= \frac{|\mathcal{I}(j)|}{\pi_{jk}} - \sum_{\substack{l\in\mathcal{J}\\ l\neq j}} \sum_{i\in\mathcal{I}(l)} \frac{G_{ijk}}{\sum_{\substack{j'\in\mathcal{J}\\ j'\neq l}} \pi_{j'k}G_{ij'k} + N_0} - \eta p^1_j\end{aligned}$$

We note that $\nabla_{\pi_{jk}}(R_2(\boldsymbol{\pi}) - \eta P(\boldsymbol{\pi})) = (1/\pi_{jk})\nabla_{\tilde{\pi}_{jk}}(R_2(\tilde{\boldsymbol{\pi}}) - \eta P(\tilde{\boldsymbol{\pi}}))$ and the logarithmic change scales the gradient by π_{jk}. Thus, the gradient iterations can be conducted in either $\boldsymbol{\pi}$ or $\tilde{\boldsymbol{\pi}}$ domain, and we can write:

$$\pi_{jk}(t+1) = \pi_{jk}(t) + \delta(t)g(t),$$

where,

$$g(t) = \nabla_{\pi_{jk}}(R_2(\boldsymbol{\pi}) - \eta P(\boldsymbol{\pi})).$$

For the solution space in $\boldsymbol{\pi}$, the feasible set of power allocations is a simplex defined by constraints (12b) and (12c). The projection of the subgradient on the simplex is straightforward and performed according to the algorithm in [42]. The convergence of the gradient-based optimization is guaranteed and proved in [43].

Note that the subgradient method can be implemented in a semi-distributed way by harnessing the X2 interface. In fact, we can write:

$$g(t) = \frac{|\mathcal{I}(j)|}{\pi_{jk}(t)} - \eta p^1_j - \sum_{\substack{l\in\mathcal{J}\\ l\neq j}} \sum_{i\in\mathcal{I}(l)} \frac{G_{ijk}}{\sum_{\substack{j'\in\mathcal{J}\\ j'\neq l}} \pi_{j'k}(t)G_{ij'k} + N_0}.$$

Simplifying the equation and using the definition of the SINR ρ_{ijk} given in (3), we obtain:

$$g(t) = \frac{|\mathcal{I}(j)|}{\pi_{jk}(t)} - \eta p^1_j - \sum_{\substack{l\in\mathcal{J}\\ l\neq j}} \sum_{i\in\mathcal{I}(l)} G_{ijk} \frac{\rho_{ilk}(t)}{G_{ilk}\pi_{lk}(t)}. \tag{17}$$

For BS j, the last term in equation (17) contains the opposite of the SINR of RB k experienced by other BSs, which is unknown to BS j. Hence, at each iteration, any BS $l \in \mathcal{J}$ must convey to its neighbors through the X2 interface a vector containing the experienced SINR on all RBs, normalized by $G_{ilk}\pi_{lk}$.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSAC.2016.2611847, IEEE Journal on Selected Areas in Communications

C. Centralized Algorithmic Solution for the Energy Efficiency Problem

Algorithm 2 details the computation process for solving the energy efficiency problem. The algorithm starts by computing the optimal solution of the per-cell scheduling introduced in Section VI-A (Line 1). Then, given an initial power allocation on RBs, the algorithm iteratively proceeds in order to solve the multi-cell power control problem.

Algorithm 2 Centralized Algorithm of the Energy Efficiency Problem

Require: $\mathcal{J}, \mathcal{I}(j), \mathcal{K}, G_{ijk}$
Require: Maximum tolerance $\epsilon \geq 0$, iteration $n \leftarrow 0$, initial value η^0
1: $\forall i \in \mathcal{I}(j), \forall k \in \mathcal{K}, \theta^*_{ik} = \frac{1}{|\mathcal{K}|}$ ▷ Optimal scheduling
2: $\pi_{jk}(0) \leftarrow p^{min}, \forall j \in \mathcal{J}, \forall k \in \mathcal{K}$
3: **repeat**
4: $\quad \eta \leftarrow \eta^n$
5: $\quad$ Iteration $t \leftarrow 0$
6: $\quad$ **repeat** ▷ Solve $(\hat{\mathcal{P}}_2(\boldsymbol{\pi}))$ in (12)
7: $\quad\quad$ **for** $j \in \mathcal{J}$ **do**
8: $\quad\quad\quad$ **for** $k \in \mathcal{K}$ **do**
9: $\quad\quad\quad\quad \pi_{jk}(t+1) \leftarrow \pi_{jk}(t) + \delta(t)(\frac{|\mathcal{I}(j)|}{\pi_{jk}(t)} - \eta p^1_j - \sum_{\substack{l \in \mathcal{J} \\ l \neq j}} \sum_{i \in \mathcal{I}(l)} (G_{ijk} \frac{\rho_{ilk}(t)}{G_{ilk}\pi_{lk}(t)}))$
10: $\quad\quad\quad\quad \pi^p_{jk}(t+1) \leftarrow$ Projection$(\pi_{jk}(t+1))$
11: $\quad\quad\quad$ **end for**
12: $\quad\quad$ **end for**
13: $\quad\quad t \leftarrow t+1$
14: $\quad$ **until** $|\boldsymbol{\pi}^p(t+1) - \boldsymbol{\pi}^p(t)| \leq \epsilon$
15: $\quad \boldsymbol{\pi}^n \leftarrow \boldsymbol{\pi}^p(t+1)$
16: $\quad \boldsymbol{\theta}^n \leftarrow \boldsymbol{\theta}^*$
17: $\quad F^n \leftarrow R(\boldsymbol{\pi}^n, \boldsymbol{\theta}^n) - \eta P(\boldsymbol{\pi}^n)$
18: $\quad \eta^{n+1} \leftarrow \frac{R(\boldsymbol{\pi}^n, \boldsymbol{\theta}^n)}{P(\boldsymbol{\pi}^n)}$
19: $\quad n \leftarrow n+1$
20: $\quad \pi_{jk}(0) \leftarrow \pi^n_{jk}, \forall j \in \mathcal{J}, \forall k \in \mathcal{K}$
21: **until** $F^n \leq \epsilon$

In each iteration of the Dinkelbach method, the projected subgradient is applied on the RBs in all BSs (Lines 7-12). The subgradient iterations are performed in $\boldsymbol{\pi}$ domain and the projection on the simplex (Line 10) is done according to the algorithm in [42]. The multi-cell power control problem converges when the variation in the power allocation between two successive iterations is less than a predefined maximum tolerance (Line 14). In practice, for realistic scenarios considered in section IX devoted for numerical results, the convergence of the projected subgradient is attained in a reasonable number of iterations. In Line 17, a new value of the objective function for the energy efficiency problem is computed. Then, a new Dinkelbach parameter is deduced (Line 18) and the iterations are repeated. The Dinkelbach method is guaranteed to converge to the optimal energy efficiency since the inner problem is solved to optimality. For realistic scenarios considered in section IX, the convergence of the Dinkelbach method is attained in a relatively small number of iterations.

VII. Distributed Formulation of the Energy Efficiency Problem

In this section, we devise a distributed joint scheduling and power control approach where BSs maximize their energy efficiency autonomously. Non-Cooperative game theory models the interactions between players competing for common resources. Here, BSs are the decision makers or players of the game that seek selfishly to maximize their own energy efficiency. BSs are assumed to make their decisions without knowing the decisions of each other.

A. The Game Formulation

We define a multi-player game $\mathcal{G}$ between the BSs. The formulation of this non-cooperative game $\mathcal{G} = \langle \mathcal{J}, S, U \rangle$ can be described as follows:

- A finite set of BSs $\mathcal{J}$;
- An action of BS j is the amount of power π_{jk} allocated on RB k, the strategy chosen by BS j is then $\boldsymbol{\pi}_j = (\pi_{j1}, ..., \pi_{j|\mathcal{K}|})$. A *strategy profile* $\boldsymbol{\pi} = (\boldsymbol{\pi}_1, ..., \boldsymbol{\pi}_{|\mathcal{J}|})$ specifies the strategies of all players;
- For each BS j, the space of pure strategies is S_j given by what follows:
$$S_j = \{\boldsymbol{\pi}_j \in \mathbb{R}^{|\mathcal{K}|}, \text{such as} \sum_{k \in \mathcal{K}} \pi_{jk} \leq p^{max}_j \text{ and } \pi_{jk} \geq p^{min}, \forall k \in \mathcal{K}\}$$
and $S = S_1 \times ... \times S_{|\mathcal{J}|}$ is the set of all strategies;
- A set of objective functions U that quantify players' objective for a given strategy profile $\boldsymbol{\pi}$. We denote $U = (U_1(\boldsymbol{\pi}), U_2(\boldsymbol{\pi}), ..., U_{|\mathcal{J}|}(\boldsymbol{\pi}))$, where U_j is the objective function of any BS j.

As BSs seek selfishly to maximize their energy efficiency, we define the objective of BS j as the ratio between its utility and its power consumption, which is given by:

$$U_j(\boldsymbol{\pi}_j, \boldsymbol{\pi}_{-j}) = \frac{\sum_{k \in \mathcal{K}, i \in \mathcal{I}(j)} \log(\frac{\theta^*_{ik}\pi_{jk}G_{jk}}{N_0 + \sum_{j' \neq j} \pi_{j'k} G_{ij'k}})}{\sum_{k \in \mathcal{K}} p^1_j \pi_{jk} + p^0_j}, \quad (18)$$

where $\boldsymbol{\pi}_{-j}$ denotes the vector of strategies played by all BSs except BS j. Note that $\boldsymbol{\theta}^*$ is the optimal solution of the per-cell scheduling problem that is computed beforehand according to Theorem 6.1.

B. The Game Properties

In a non-cooperative game, an efficient solution is obtained when all players adhere to a NE. A NE is a profile of strategies in which no player will profit from deviating its strategy unilaterally. Hence, it is a strategy profile where each player's strategy is an optimal response to the other players' strategies.

$$U_j(\boldsymbol{\pi}_j, \boldsymbol{\pi}_{-j}) \leq U_j(\boldsymbol{\pi}'_j, \boldsymbol{\pi}_{-j}), \forall j \in \mathcal{J}, \forall \boldsymbol{\pi}'_j \in S_j. \quad (19)$$

According to the work in [44], a NE exists for our game because $U_j(\boldsymbol{\pi}_j, \boldsymbol{\pi}_{-j})$ is a quasi-concave function in $\boldsymbol{\pi}_j$ (it is the ratio of two concave functions) and is continuous in $\boldsymbol{\pi}_{-j}$. A NE is a static concept that often abstracts away the question of how it is reached. Thus, the main challenge in non-cooperative

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game theory is to devise practical algorithms to reach those equilibriums. The simplest example of such algorithms are repeated Best Response dynamics: each player selects the best (locally optimal) response to other players' strategies, until convergence. However, convergence of repeated Best Response is not guaranteed in general. In this work, we are in presence of a type of games called super-modular games where a greedy Best Response algorithm permits attaining NEs. In the following, we introduce a formal definition of super-modular games and prove that our distributed joint scheduling and power control game $\mathcal{G}$ belongs to the latter class.

According to [45], $\mathcal{G}$ is super-modular if for any BS $j \in \mathcal{J}$:

1) the strategy space S_j is a compact sub-lattice of $\mathbb{R}^{|\mathcal{K}|}$;
2) the objective function U_j is super-modular, that is $\frac{\partial U_j}{\partial \pi_l \partial \pi_j} \geq 0$, $\forall l \in \mathcal{J} - \{j\}$, $\forall \boldsymbol{\pi}_j \in S_j$, and $\forall k \in \mathcal{K}$.

In [45], [46], proof is given for the following two results in a super-modular game:

- if each BS j either initially uses its lowest or largest policy in S_j, then a Best Response algorithm converges monotonically to a NE (that may depend on the initial state);
- if we start with a feasible policy, then the sequence of best responses monotonically converges to a NE: it monotonically increases in all components in the case of maximization in a super-modular game.

Proposition 7.1: Game $\mathcal{G}$ is a super-modular game.

Proof: To prove the super-modularity of the game, we need to verify the aforementioned conditions. First, the strategy space S_j is obviously a compact convex set of $\mathbb{R}^{|\mathcal{K}|}$. Hence, it suffices to verify the super-modularity of the objective function of any BS j as there are no constraint policies for $\mathcal{G}$:

$$\frac{\partial U_j}{\partial \pi_{lk} \partial \pi_{jk}} = \frac{p_j^1}{(\sum_{k \in \mathcal{K}} p_j^1 \pi_{jk} + p_j^0)^2} \times (\sum_{i \in \mathcal{I}(j)} \frac{G_{ilk}}{N_0 + \sum_{j' \neq j} \pi_{j'k} G_{ij'k}}) \geq 0, \quad \forall l \in \mathcal{J} - \{j\}, \forall k \in \mathcal{K}.$$

□

VIII. Solving the Distributed Energy Efficiency Problem

Following the formulation of the distributed energy efficiency problem, we focus in this section on the algorithmic solutions. As for the centralized formulation, we make use of the Dinkelbach method associated with a projected subgradient. We also provide the formal proof for two valuable properties of the non-cooperative game at hand, namely the existence and the uniqueness of the NE.

A. Computing the NE

As we proved that we are in presence of a super-modular game, we implement a Best Response algorithm to reach its pure NEs. Accordingly, at each iteration t, BS j strives to find the following optimal power level as a response to $\boldsymbol{\pi}_{-j}(t-1)$:

$$\boldsymbol{\pi}_j^*(t) = \underset{\boldsymbol{\pi}_j}{\operatorname{argmax}}\, U_j(\boldsymbol{\pi}_j, \boldsymbol{\pi}_{-j}), \text{subject to: } \boldsymbol{\pi}_j \in S_j, \quad (20)$$

which amounts to the following optimization problem:

$$\max_{\boldsymbol{\pi}_j} U_j(\boldsymbol{\pi}_j, \boldsymbol{\pi}_{-j}) \quad (21a)$$

$$\text{subject to} \quad \sum_{k \in \mathcal{K}} \pi_{jk} \leq p_j^{max}, \quad (21b)$$

$$\pi_{jk} \geq p^{min}, \ \forall k \in \mathcal{K}. \quad (21c)$$

1) Projected subgradient-based algorithm: Problem (21) cannot be solved in a straightforward manner as it is non-convex. However, as it is a fractional problem, an optimal solution is obtained by iteratively solving the parametrized convex problem (22), according to the Dinkelbach method [13].

$$(\hat{\mathcal{P}}_3(\boldsymbol{\pi})): \quad \max_{\boldsymbol{\pi}_j} F_j(\eta_j) = \sum_{k \in \mathcal{K}, i \in \mathcal{I}(j)} \log(\frac{\theta_{ik}^* \pi_{jk} G_{ijk}}{N_0 + \sum_{j' \neq j} \pi_{j'k} G_{ij'k}}) - \eta_j (\sum_{k \in \mathcal{K}} p_j^1 \pi_{jk} + p_j^0) \quad (22a)$$

$$\text{subject to} \quad \sum_{k \in \mathcal{K}} \pi_{jk} \leq p_j^{max}, \quad (22b)$$

$$\pi_{jk} \geq p^{min}, \ \forall k \in \mathcal{K}. \quad (22c)$$

We may intuitively take η_j as the *price* of the total system power of BS j which results in $\max F_j = 0$ at the optimal value of η_j. The Dinkelbach method produces a decreasing sequence of η_j values, which converges to the optimal value at a super-linear convergence rate. The convergence is guaranteed if the inner problem (22) is solved to optimality.

In order to solve problem $(\hat{\mathcal{P}}_3(\boldsymbol{\pi}))$ for each BS $j \in \mathcal{J}$, we start by transforming it into a convex optimization via a change of variables $\tilde{\pi}_{jk} = \log(\pi_{jk})$. Similarly to the computations in Section VI-B2, we can have recourse to the subgradient method for constrained optimization and solve it in a distributed way. This algorithm takes the form:

$$\tilde{\pi}_{jk}(t+1) = \tilde{\pi}_{jk}(t) + \delta(t)\tilde{g}(t),$$

where t is the iteration number, $\delta(t)$ a step size, and $\tilde{g}(t)$ a subgradient of the objective function of problem $(\hat{\mathcal{P}}_3(\boldsymbol{\pi}))$:

$$\tilde{g}(t) = \nabla_{\tilde{\pi}_{jk}} F_j(\eta_j) = |\mathcal{I}(j)| - \eta_j p_j^1 \pi_{jk}(t).$$

Coming back to the solution space in $\boldsymbol{\pi}$ instead of $\tilde{\boldsymbol{\pi}}$, the derivative of the objective function in (22a) with respect to π_{jk} is given by:

$$g(t) = \nabla_{\pi_{jk}} F_j(\eta_j) = \frac{|\mathcal{I}(j)|}{\pi_{jk}(t)} - \eta_j p_j^1.$$

We note that $g(t) = \tilde{g}(t)/\pi_{jk}$ and the logarithmic change scales the gradient by π_{jk}. Thus, the gradient iterations can be conducted in either $\boldsymbol{\pi}$ or $\tilde{\boldsymbol{\pi}}$ domain, and we can write:

$$\pi_{jk}(t+1) = \pi_{jk}(t) + \delta(t)g(t).$$

Once again, the feasible set of power allocations is a simplex and the projection of the subgradient is straightforward and performed according to the algorithm in [42].

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2) The power expression at equilibrium: The optimum $\boldsymbol{\pi}^*$ of the convex problem (22) must satisfy the Karush-Kuhn-Tucker (KKT) conditions, *i.e.,* there exists a unique Lagrange multiplier $\beta \geq 0$ such that:

$$\nabla_{\pi_{jk}}(F_j(\eta_j)) + \beta \nabla_{\pi_{jk}}(f_j(\boldsymbol{\pi}_j)) = 0, \; \forall k \in \mathcal{K}, \tag{23a}$$

$$\beta f_j(\boldsymbol{\pi}_j) = 0, \tag{23b}$$

$$\pi_{jk} \geq p^{min}, \; \forall k \in \mathcal{K}, \tag{23c}$$

where $f_j(\boldsymbol{\pi}_j) = p_j^{max} - \sum_{k \in \mathcal{K}} \pi_{jk}$. Thus, according to (23a), the power allocation is given by:

$$\pi_{jk} = \frac{|\mathcal{I}(j)|}{p_j^1 \eta_j + \beta}, \; \forall k \in \mathcal{K}. \tag{24}$$

Note that all power levels for a given BS j are equal at equilibrium. Finally, to obtain the power levels that are sought for, we have recourse to (23b): if $\beta > 0$, $\sum_{k \in \mathcal{K}} \pi_{jk} = p_j^{max}$ at optimality and hence, by virtue of the equality among the power components, we have $\pi_{jk} = p_j^{max}/|\mathcal{K}|, \forall k \in \mathcal{K}$. Otherwise, if $\beta = 0$, $\pi_{jk} = |\mathcal{I}(j)|/(\eta_j p_j^1), \forall k \in \mathcal{K}$. Hence, we deduce the following:

$$\pi_{jk}(t) = \min(\frac{|\mathcal{I}(j)|}{p_j^1 \eta_j(t)}, \frac{p_j^{max}}{|\mathcal{K}|}), \; \forall k \in \mathcal{K}, \forall j \in \mathcal{J}. \tag{25}$$

B. Uniqueness of the NE

The goal of the present section is to find under what conditions the game $\mathcal{G}$ converges to a unique NE, which is a valuable result. In fact, one of the difficulties that has limited the usefulness of the concept of NE in non-cooperative game theory is the lack of uniqueness of such equilibriums. In fact, the majority of games possess a plethora of NEs as shown in [47].

Assume that the game $\mathcal{G}$ admits two distinct NE points, denoted by $\boldsymbol{\pi}_j^{(r)}$ where $r \in \mathcal{R} = \{0, 1\}$ for any BS j which are optimum points of the optimization problem (22) and hence satisfy the KKT conditions in (23). Moreover, as the functions $f_j(\boldsymbol{\pi}_j^{(r)})$ are (linear) convex, we have for each $\boldsymbol{\pi}_j^{(r)}$:

$$f_j(p) - f_j(\boldsymbol{\pi}_j^{(r)}) \geq (p - \boldsymbol{\pi}_j^{(r)}) \nabla_{\tilde{\pi}_{jk}}(f_j(\boldsymbol{\pi}_j^{(r)})), \quad \forall k \in \mathcal{K}, \forall j \in \mathcal{J}, \tag{26}$$

where $p = \boldsymbol{\pi}_j^{(1)}$ resp. $(\boldsymbol{\pi}_j^{(0)})$ for $r = 0$ (resp. $r = 1$). Multiplying the equation in (23a) by $(\pi_{jk}^{(0)} - \pi_{jk}^{(1)})$ for $r = 0$ and by $(\pi_{jk}^{(1)} - \pi_{jk}^{(0)})$ for $r = 1$, $\forall k \in \mathcal{K}$, and summing over r, we obtain $\forall j \in \mathcal{J}$:

$$\begin{aligned} 0 = &(\pi_{jk}^{(1)} - \pi_{jk}^{(0)}) \nabla_{\pi_{jk}}(f_j(\boldsymbol{\pi}_j^{(0)})) \\ &+ (\pi_{jk}^{(0)} - \pi_{jk}^{(1)}) \nabla_{\pi_{jk}}(f_j(\boldsymbol{\pi}_j^{(1)})) \\ &+ \beta^{(0)}(\pi_{jk}^{(1)} - \pi_{jk}^{(0)}) \nabla_{\pi_{jk}}(f_j(\boldsymbol{\pi}_j^{(0)})) \\ &+ \beta^{(1)}(\pi_{jk}^{(0)} - \pi_{jk}^{(1)}) \nabla_{\pi_{jk}}(f_j(\boldsymbol{\pi}_j^{(1)})). \end{aligned} \tag{27}$$

The last two expressions in (27) are upper bounded by:

$$\beta^{(0)}(f_j(\boldsymbol{\pi}_j^{(1)}) - f_j(\boldsymbol{\pi}_j^{(0)})) + \beta^{(1)}(f_j(\boldsymbol{\pi}_j^{(0)}) - f_j(\boldsymbol{\pi}_j^{(1)})). \tag{28}$$

As $\beta^{(r)} f_j(\pi_j^{(r)}) = 0$, (28) is equivalent to:

$$\beta^{(0)} f_j(\boldsymbol{\pi}_j^{(1)}) + \beta^{(1)} f_j(\boldsymbol{\pi}_j^{(0)}). \tag{29}$$

We can induce the uniqueness of the NE in two scenarios $\forall r \in \mathcal{R}, \forall j \in \mathcal{J}$:

- when the sum of allocated power to RBs is maximal *i.e.,* $f_j(\pi_j^{(r)}) = 0$, deemed the high interference case;
- and in the converse scenario when the sum of allocated power to RBs does not reach p_j^{max} *i.e.,* $\beta^{(r)} = 0$, deemed the low interference case.

In both cases, the expression in (29) is null, which leads to the following:

$$(\pi_{jk}^{(1)} - \pi_{jk}^{(0)})(\nabla_{\pi_{jk}}(f_j(\boldsymbol{\pi}_j^{(0)})) - \nabla_{\pi_{jk}}(f_j(\boldsymbol{\pi}_j^{(1)}))) \geq 0, \quad \forall k \in \mathcal{K}. \tag{30}$$

In particular, for the high interference case, $f_j(\boldsymbol{\pi}_j^{(r)}) = 0$ leads to $\pi_{jk}^{(r)} = p_j^{max}/|\mathcal{K}|, \forall r \in \mathcal{R}, \forall j \in \mathcal{J}$, which is obviously a unique NE. For the low interference case, we resort to (30) which re-writes as follows:

$$\begin{aligned} &(\pi_{jk}^{(1)} - \pi_{jk}^{(0)})(\frac{1}{\pi_{jk}^{(0)}} - \eta_j^{(0)} p_j^1 - \frac{1}{\pi_{jk}^{(1)}} + \eta_j^{(1)} p_j^1) = \\ &\qquad \frac{(\pi_{jk}^{(1)} - \pi_{jk}^{(0)})^2 (1 - |\mathcal{I}|)}{\pi_{jk}^{(0)} \pi_{jk}^{(1)}} \leq 0, \forall k \in \mathcal{K}. \end{aligned} \tag{31}$$

Thus, (30) or equivalently (31) can only equate to zero. Consequently $\pi_{jk}^{(1)} = \pi_{jk}^{(0)}$, $\forall k \in \mathcal{K}, \forall j \in \mathcal{J}$.

C. Distributed Algorithmic Solution of the Energy Efficiency Problem

Algorithm 3 details the computation process for solving the energy efficiency problem in a distributed manner. The algorithm starts by computing the optimal solution of the per-cell scheduling policy introduced in Section VI-A (Line 1). Then, given an initial power allocation on RBs, the Best Response algorithm proceeds in rounds until convergence to a NE (Line 23). The results shown in Section IX reveal that a NE is attained in a small number of rounds. Each round consists of performing power allocation successively on all BSs in the network. When dealing with BS j, the optimal power allocation is computed by the Dinkelbach method (Lines 5-21). In each iteration of the Dinkelbach method, the projected subgradient is applied on the RBs of BS j (Line 8-14). The power control problem converges when the variation in the power allocation between two successive iterations is less than a predefined maximal tolerance (Line 14). In Line 17, a new value of the objective function of the energy efficiency problem for BS j is computed. Then, a new Dinkelbach parameter is deduced (Line 18) and the iterations are repeated. For realistic scenarios considered in Section IX, the convergence of the Dinkelbach method is attained in a small number of iterations.

IX. Numerical Results

We consider an LTE network of seven hexagonal cells and 10 users uniformly distributed per BS. The inter-BS distance equals 500 m as in an urban environment. The system bandwidth equals 3 MHz and 15 RBs are available for each BS. Channels are generated using the publicly-available MATLAB

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Algorithm 3 Distributed solution for the energy efficiency problem

Require: $\mathcal{J}, \mathcal{I}(j), \mathcal{K}, G_{ijk}$
Require: Maximum tolerance $\epsilon \geq 0$, iteration $n \leftarrow 0$, η^0
$\forall i \in \mathcal{I}(j), \forall k \in \mathcal{K}, \theta^*_{ik} = \frac{1}{|\mathcal{K}|}$ ▷ Optimal scheduling
$\pi_{jk}(0) \leftarrow p^{min}, \forall j \in \mathcal{J}, \forall k \in \mathcal{K}$
repeat
  for $j \in \mathcal{J}$ **do**
    repeat
      $\eta_j \leftarrow \eta^n_j$
      Step $t \leftarrow 0$
      repeat ▷ Solve $(\hat{\mathcal{P}}_2(\boldsymbol{\pi}))$ in (12)
        for $k \in \mathcal{K}$ **do**
          $\pi_{jk}(t+1) \leftarrow \pi_{jk}(t) + \delta(t)(\frac{|\mathcal{I}(j)|}{\pi_{jk}(t)} - \eta_j p^1_j)$
          $\pi^p_{jk}(t+1) \leftarrow$ Projection$(\pi_{jk}(t+1))$
        end for
        $t \leftarrow t+1$
      until $|\boldsymbol{\pi}^p_j(t+1) - \boldsymbol{\pi}^p_j(t)| \leq \epsilon$
      $\boldsymbol{\pi}^n_j \leftarrow \boldsymbol{\pi}^p_j(t+1)$
      $\boldsymbol{\theta}^n_j \leftarrow \boldsymbol{\theta}^*_j$
      $F^n_j \leftarrow R(\boldsymbol{\pi}^n_j, \boldsymbol{\theta}^n_j) - \eta_j P(\boldsymbol{\pi}^n_j)$
      $\eta^{n+1}_j \leftarrow \frac{R(\boldsymbol{\pi}^n_j, \boldsymbol{\theta}^n_j)}{P(\boldsymbol{\pi}^n_j)}$
      $n \leftarrow n+1$
      $\pi_{jk}(0) \leftarrow \pi^n_{jk}, \forall j \in \mathcal{J}, \forall k \in \mathcal{K}$
    until $F^n_j \leq \epsilon$
  end for
until $|\boldsymbol{\pi}^p(t+1) - \boldsymbol{\pi}^p(t)| \leq \epsilon$

implementation of the WINNER Phase II Channel Model [48]. The shadow fading map follows a lognormal-distributed 2D space-correlated, as in [49]: the Gaussian random variable has a zero mean and a standard deviation of 10 dB. We consider the following numerical parameters related to the power of each BS $j \in \mathcal{J}$ [38]:

- The transmit power scaling coefficient p^1_j introduced in (1) equals 4.7.
- The transmit power independent coefficient p^0_j equals 130.
- The maximal power consumption p^{max}_j equals 20 W (or equivalently 43 dBm).
- The minimum power on each RB p^{min} equals 0.1 W.

Numerical results are obtained for 30 runs of each algorithm. Users are generated according to a random uniform distribution (leading to moderate interference), except in Sections IX-C1 and IX-C2 dedicated to high and low interference scenarios. In these sections, the distance between users and the associated BS follows a Gaussian distribution with a standard deviation of 75 m, centered at the BS for the low interference scenario or at 100 m for the high interference scenario.

A. Centralized and Distributed Approaches

The following results allow us to compare the centralized and the distributed approaches of the energy efficiency problem. Recall that the centralized approach is solved using a projected subgradient algorithm, while the distributed approach, formulated as a super-modular game, is solved using a Best Response algorithm. Both approaches make use of the per-cell scheduling given in Theorem 6.1. Energy efficiency is computed according to the objective function in (6a).

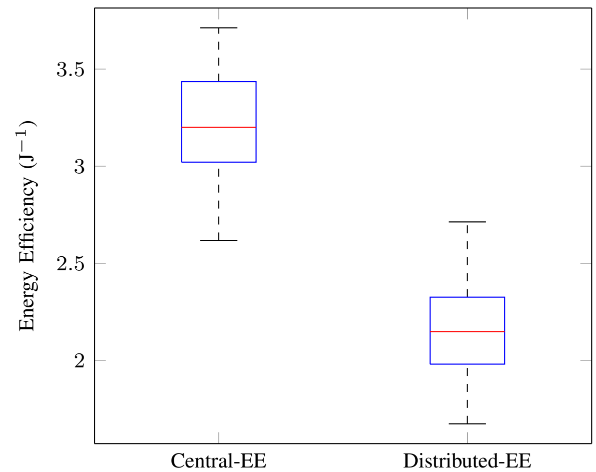

Fig. 1. Energy efficiency of the centralized and distributed approaches

In Fig. 1, we represent a boxplot of the energy efficiency for the centralized (Central-EE) and distributed (Distributed-EE) approaches. On each boxplot, the central mark is the median, the edges of the box are the 25th and 75th percentiles, the whiskers extend to the most extreme data points which are not considered outliers, and outliers are plotted individually. In our work, the energy efficiency is computed as the ratio between a unitless network utility (sum of logarithms of SINRs) and a total power consumption in Joule. Therefore, the energy efficiency is expressed in J^{-1} units. We note that the centralized approach achieves higher energy efficiency as expected. Precisely, the comparison criterion corresponds to the objective of the optimization problem. Thus, the computed solution corresponds to the highest achievable value. In the distributed approach, each BS strives to maximize its own energy efficiency. However, this selfish behavior does not lead to maximizing the sum of the energy efficiency as with the optimal centralized approach. This is commonly known as the *price of anarchy* [50].

In Fig. 2(a), we plot the power distribution on all RBs in the network, and in Fig. 2(b), we portray the SINR distribution. For the former, we note that the optimal power distribution computed by the centralized approach has a median value of 0.1 and a very small standard deviation. However, the optimal power computed by the distributed approach has a median value of approximatively 1.0 and a larger standard deviation. In fact, the centralized approach allocates the minimum power on each RB: this ensures low interference, low power consumption, and achieves the highest energy efficiency given by the ratio of total SINR and total power consumption; whereas in the distributed approach, the BSs allocate higher power on RBs in order to selfishly increase their energy efficiency neglecting the harmful impact of interference on neighboring cells. Note that the power consumption in the denominator of the objective function of each BS prevents from allocating the maximal power on each RB in the distributed approach.

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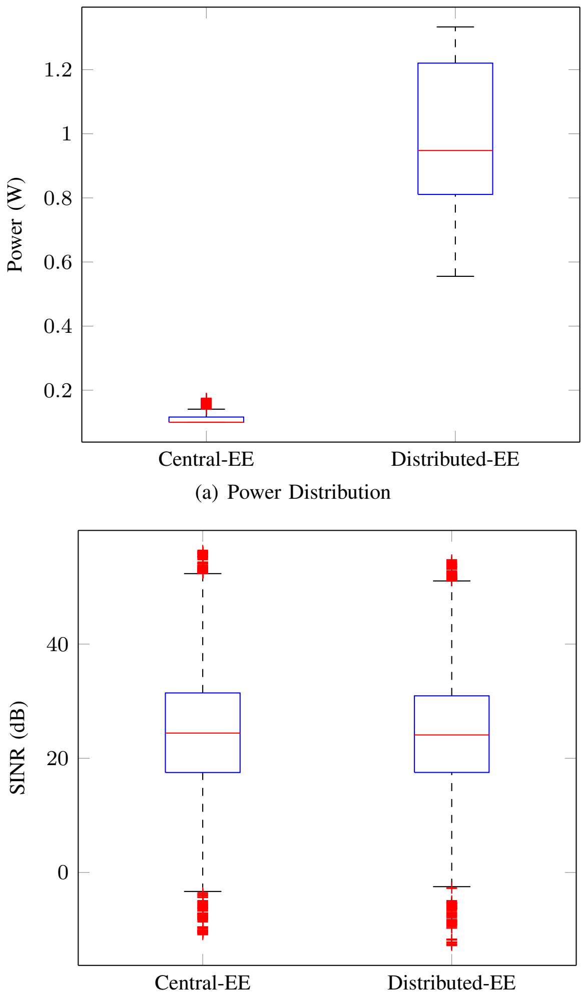


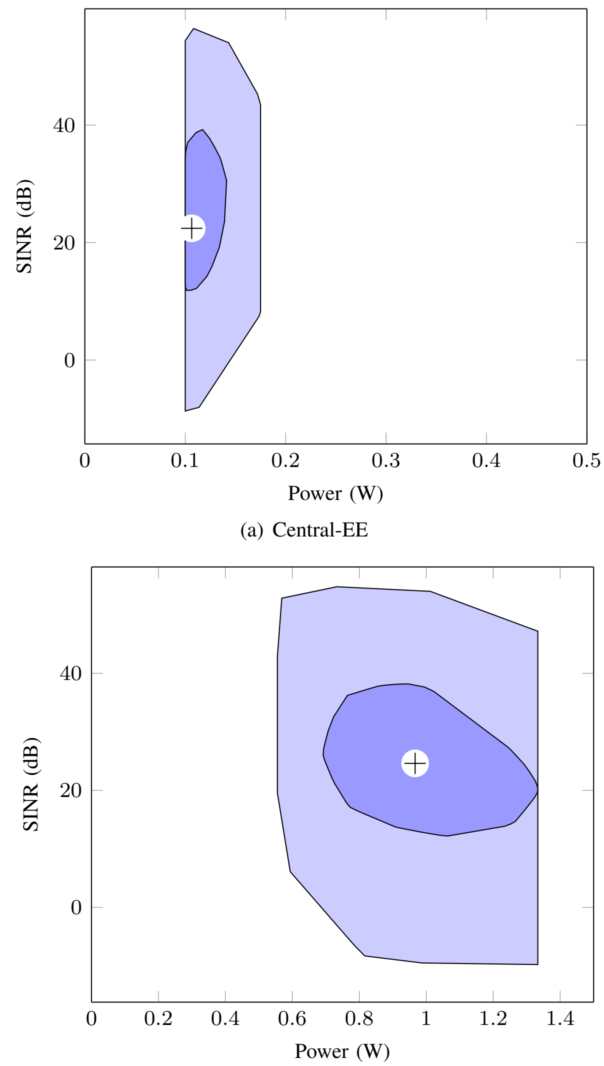
(a) Power Distribution

(b) SINR Distribution

Fig. 2. Power and SINR distribution of the centralized and distributed approaches

The power distribution becomes more significant when coupled with the SINR distribution in Fig. 2(b). Precisely, the centralized and distributed approaches provide similar SINR for the users in the network. The median value is close to 25 dB and the percentiles between 0 dB and 50 dB. This simulation results reveals the importance of the centralized approach that avoids unnecessary power consumption without jeopardizing the SINR of serviced users.

The bag plots in Fig. 3 are used to assess the power allocated to RBs as a function of the SINR obtained by end-users. Bagplots are equivalent of boxplots in two dimensions [51]. The bag of the centralized approach (in dark blue) is thinner than that of the distributed approach: this means that the power spread is smaller, whereas the SINR spread is equivalent in both scenarios. Furthermore, the bag slopes downward for the distributed approach. We deduce that we have negative correlation, *i.e.,* high power values lead to a deterioration of the SINR values. In fact, when selfish BSs increase inconsiderably power to improve their energy efficiency, the achieved SINR is relatively low. This downside is avoided in the centralized approach as the latter operates while considering the interference impact. We also notice that the SINR-Power bag plot for the centralized approach is very skewed as the median (marked by a cross label) lies in the left part of the bag. By contrast, the bag plot of the distributed approach is nicely balanced and its form suggests an elliptic distribution. Finally, both distributions are medium-tailed, judging from the size of the loop (in light blue) and the low number of outliers (points outside of the loop).



(a) Central-EE

(b) Distributed-EE

Fig. 3. SINR-Power bagplot of the centralized and distributed approaches

B. *Comparison with State-of-the-Art Approaches*

The following results enable to assess the performances of the approaches introduced in this work compared to the state-of-the-art. We consider four benchmark schemes:

1) Central-SE that maximizes the sum of the log of the SINR without taking into account the power consumption.
2) MaxPower where power on each RB is equal to the maximum sector power divided by the number of RBs.
3) NoInterference-EE that maximizes the energy efficiency and neglects the inter-cell interference.

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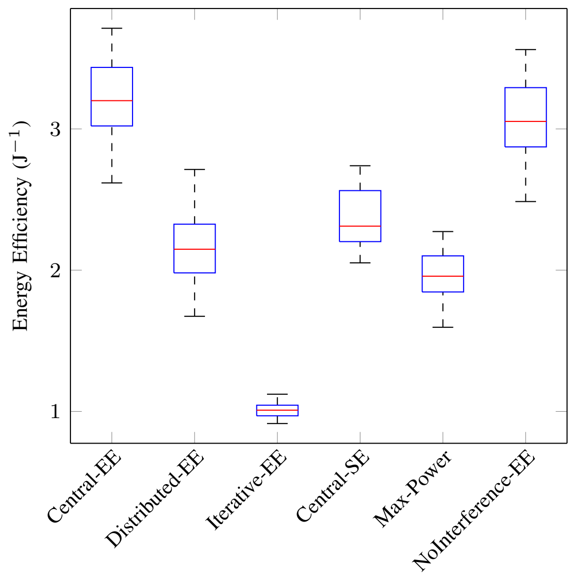
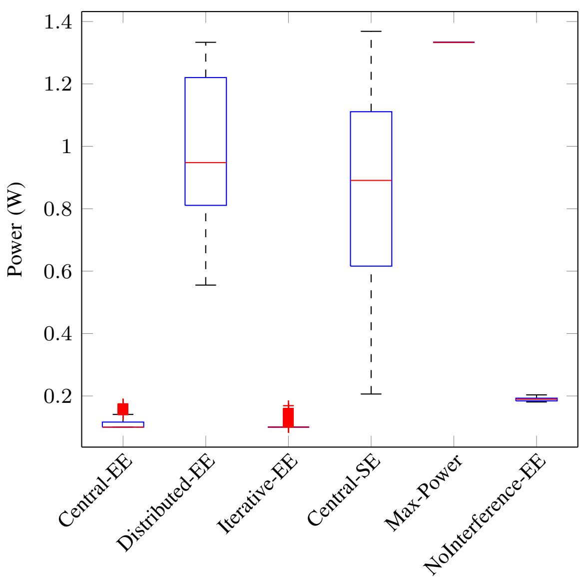


(a) Energy Efficiency



(b) Power Allocation

Fig. 4. Comparison of the centralized and distributed approaches with various heuristics

4) Iterative-EE, an adaption of the work in [24], that is detailed hereafter.

Iterative-EE is a joint approach where scheduling and power allocation are performed iteratively. In each iteration, the algorithm computes an optimal scheduling solution where each RB is allocated to the user with the highest SINR. Then, taking the optimal schedule, the algorithm computes a power allocation on RBs that maximizes the network energy efficiency. For the sake of comparison and scientific rigor, we consider the same energy efficiency objective function as in Central-EE (12a). Authors in [24] prove that the iterative algorithm converges toward a global optimal solution of the joint scheduling and power control problem.

Note that the per-cell scheduling for Central-SE, MaxPower, and NoInterference-EE is performed according to the optimal solution given in Theorem 6.1. Such scheduling contributes in achieving a proportional fair resource allocation. In contrast, scheduling in Iterative-EE tends to favor users with good radio conditions. As it is shown in the sequel, the dissimilarity between the scheduling paradigms has a significant impact on the performance results.

Let us start by analyzing the energy efficiency of the various approaches in Fig. 4(a) supported by the power distribution in Fig. 4(b). Considering the median values of energy efficiency for the diverse approaches, we see as expected that the optimal centralized approach surpasses all the others. We notice that the broadly adopted Max-Power has bad performance because of unduly power consumption. As for the NoInterference-EE approach, we note that it has relatively good performance. In fact, as the latter is oblivious to interference, it tends to lower power in order to increase the objective function. Precisely, as we are in presence of a moderate interference scenario, the Central-EE behaves similarly but has more stringent limit on power as it takes interference into account. This behavior is further highlighted in pertinent scenarios with high and low interference presented in the following section. The distributed approach has relatively low performance owing to its propensity to power consumption as already pinpointed. Regarding the Central-SE approach, the achieved energy efficiency is low and the power distribution has a median value close to 1.0 W. Precisely, power allocation in Central-SE is driven by interference minimization (in order to increase spectral efficiency) and not by power consumption reduction. Finally, we note that Iterative-EE achieves the worst performance in terms of energy efficiency despite its low power consumption. Particularly, the scheduling policy applied in Iterative-EE is biased against users with bad radio conditions. Such unfairness in the scheduling scheme reduces the logarithmic network utility computed as in (4), and consequently lowers the energy efficiency. In contrast, our optimal per-cell scheduling scheme jointly applied with the other approaches achieves proportional fairness and leads to higher energy efficiency values.

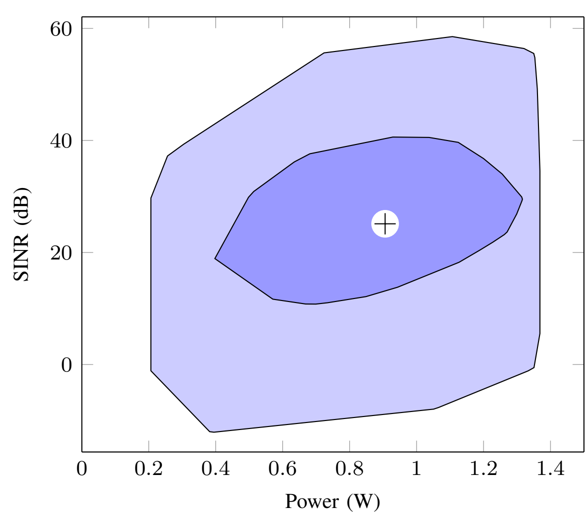


Fig. 5. SINR-Power bagplot of Central-SE

The bag plot in Fig. 5 enables to get more insights on the performance of Central-SE. The median SINR value (marked

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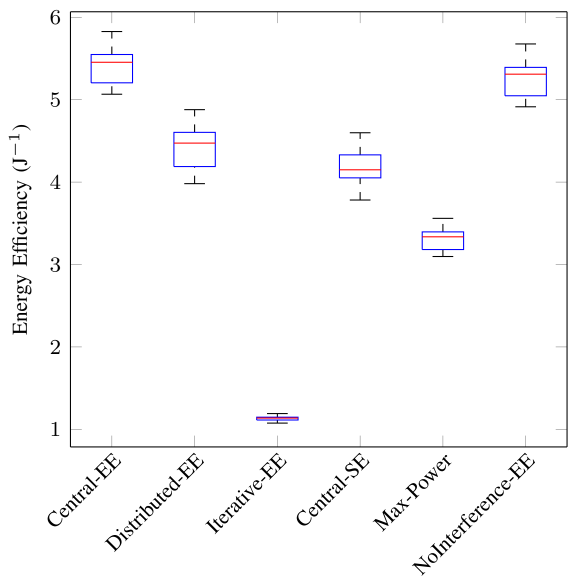
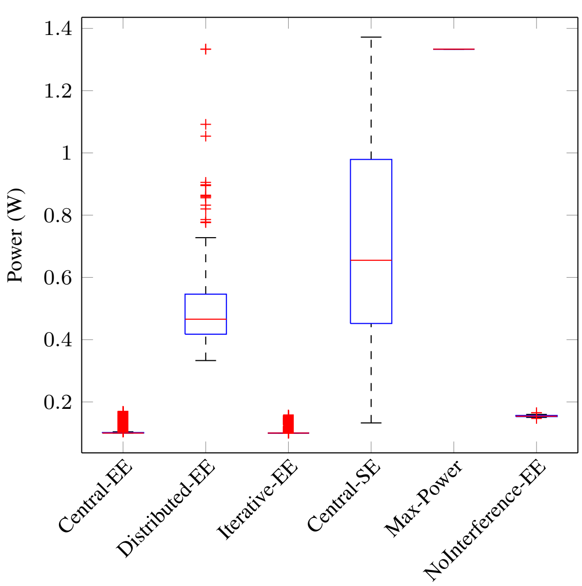
by a cross label) is very close to that of the Central-EE and Distributed-EE presented in Fig. 3. The bag spread (in dark blue) is larger in terms of power allocation and it slopes upward. This reveals a positive correlation where high power values correspond to high SINR values. Moreover, upper right side of the loop (in light blue) is higher than that of the other approaches. Central-SE achieves higher SINR for a small number of users at the expense of lower energy efficiency.



(a) Energy efficiency



(b) Power allocation

Fig. 6. Energy efficiency and power allocation in low interference scenario

C. High Interference and Low Interference Scenarios

In this section, we consider pertinent scenarios of high and low interference in order to obtain more insights on the devised approaches.

1) Low interference: In the low interference scenario sketched in Fig. 6(a), the energy efficiency of Central-EE, Distributed-EE, Central-SE, Max-Power, and NoInterference-EE approaches is increased. In presence of low interference, these methods achieve higher SINRs for each consumed power unit, compared with the moderate interference scenario. Therefore, the ratio (*i.e.*, energy efficiency) is increased while the allocated power is reduced (except for Max-Power where the power is obviously constant). As shown in Fig. 6(b), the percentiles of power distribution are now equal to the median value of 0.1 W for the centralized approach. Moreover, the median power value of the distributed approach equals 0.5 W compared to 1.0 W in case of moderate interference.

Contrary to the above-cited approaches, the energy efficiency of Iterative-EE does not increase compared with the moderate interference scenario. It has a median value close to 1 as show in Fig. 6(a). In fact, the scheduling policy implemented by Iterative-EE is biased against users with bad radio conditions. Yet, these users benefit most from the low interference scenario to increase their SINR. Consequently, the unfair scheduling used with Iterative-EE prevents it from increasing its energy efficiency in the scenario at hand. On the opposite, the scheduling applied with the remaining approaches follows a proportional fair resource allocation policy. Such a fair scheduling enables these approaches to take benefit from the SINR improvement of users with bad radio conditions. This enforces the increase in the energy efficiency of Central-EE, Distributed-EE, Central-SE, Max-Power, and NoInterference-EE as analyzed above.

2) High interference: For the same reasons explained above, the energy efficiency of the compared approaches is decreased in high interference scenario. In fact, the power allocation is increased on all RBs in order to achieve higher SINRs and counterbalances the impact of more noxious interference. As portrayed in Fig. 7(a), we notice smaller discrepancy between the energy efficiency of the diverse approaches. Here, Iterative-EE has the highest median value owing to a scheduling policy that favors users with good radio conditions. As such users are less impacted by the interference escalation, Iterative-EE outranks the remaining approaches and shows the best energy efficiency in harsh interference conditions. Moreover, the Max-Power performances are acceptable in such a case and this simplistic method can be useful in high interference scenarios. In Fig. 7(b), we note that the allocated power is higher for the diverse approaches compared to the moderate interference scenario. The median value of the centralized approach is increased and the distributed approach always allocates the maximum power value on all RBs. Here again, the latter shows high power consumption driven by selfish behavior of the BSs.

D. Convergence Rate

In the following, we assess the convergence rate of the algorithm for solving the centralized approach in a moderate interference scenario. In Fig. 8(a), we show the power on each RB as a function of the number of subgradient iterations, and in dotted vertical lines the Dinkelbach iterations. The algorithm starts with an initial value of the parameter $\eta^0 = 2$ and a power value of p^{min} on all RBs. The subgradient updates

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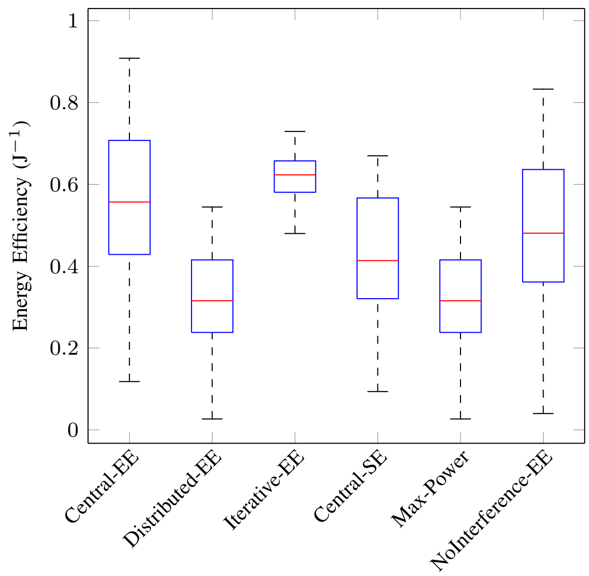
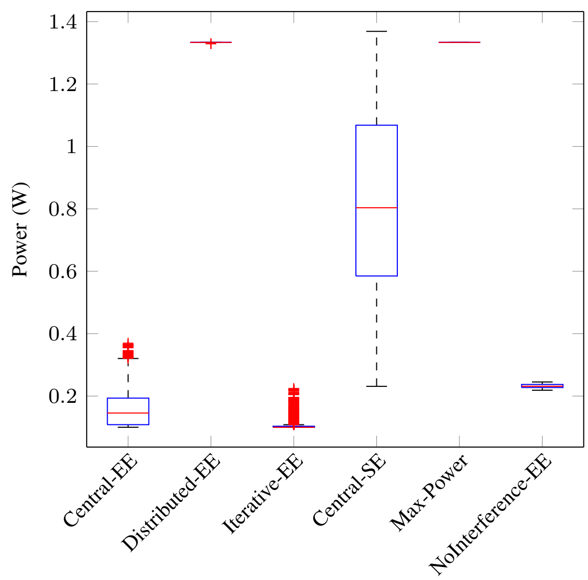
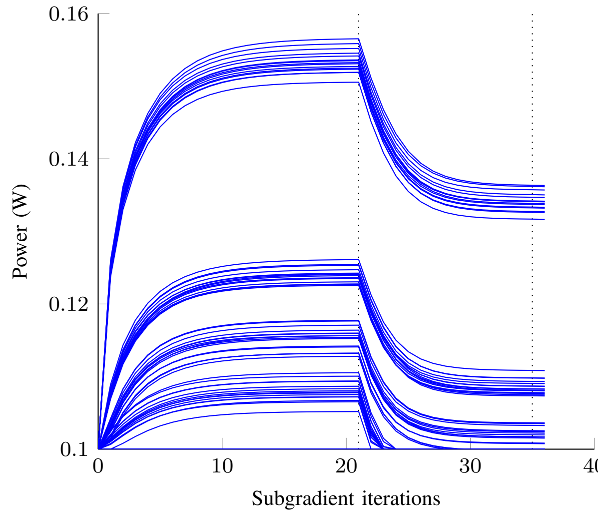


(a) Energy efficiency

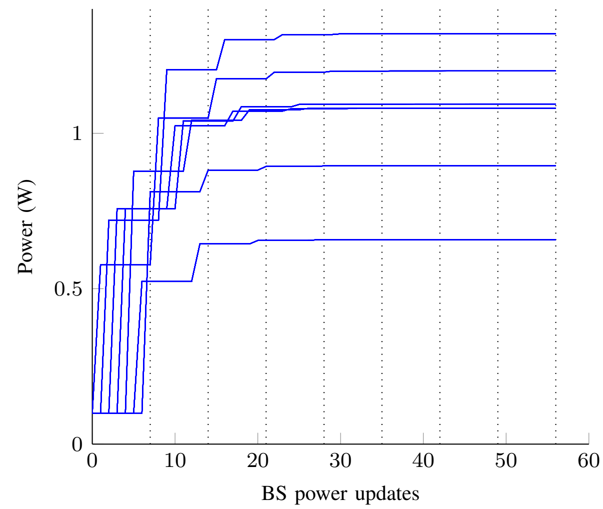


(b) Power allocation

Fig. 7. Energy efficiency and power allocation in high interference scenario



(a) Central-EE



(b) Distributed-EE

Fig. 8. Power updates for the centralized and distributed approaches

increase the power until a fast convergence in around 20 steps. In our computation, the constant step size parameter δ equals 10^{-3} and the convergence threshold ϵ equals 10^{-4}. Then, the algorithm computes a new value of the η parameter and runs the projected subgradient-based algorithm. We note that the number of Dinkelbach iterations equals 3 with a decreasing number of subgradient iterations.

In Fig. 8(b), we plot the power on each RB as a function of the number of BS power updates in the distributed approach. These updates are performed when solving the optimization problem (22) successively for each BS. The vertical dotted lines show the completion of power updates on all BSs, *i.e.*, the Best Response round. We note that for a given BS, the computed power is the same on all RBs as proven in Section VIII-A2. Starting from the minimal power value on all RBs, the BSs perform in turns local Dinkelbach procedures with subgradient iterations following Algorithm 3 and increase monotonically the allocated power. Similarly to the centralized approach, the constant step size parameter δ equals 10^{-3} and the convergence threshold ϵ equals 10^{-4}. In this example, the Best Response algorithm converges in only eight rounds.

In Fig. 9, we show the energy efficiency objective of each BS as a function of the number of BS power updates. As noted in Fig. 8(b), the NE is reached in only eight rounds, shown with vertical dotted lines. Solely in the first round, the energy efficiency exhibits large variations for each BS. This is due to the fact that all BSs start with a minimal power allocation on each RB. Interestingly, these variations diminish considerably starting from the second round until reaching the equilibrium.

In Fig. 10, we plot the distribution of the number of Best Response rounds until convergence for 30 simulation runs. The median value equals 7 and the maximum value equals 8. In the following discussion section, we stress on the fast convergence of the Best Response algorithm, which is a real asset of a distributed approach.

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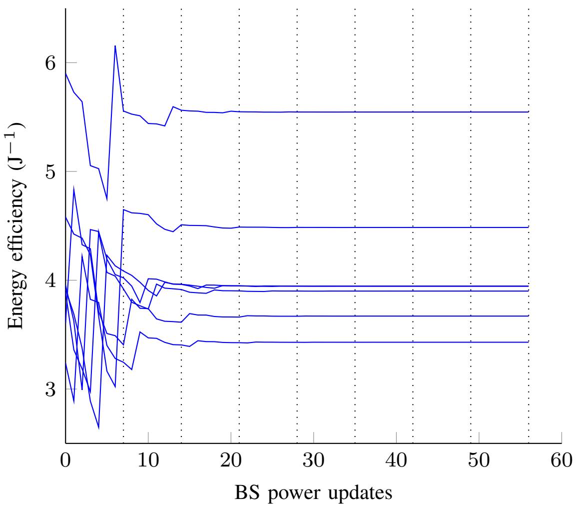

Fig. 9. Energy efficiency variation of the distributed approach

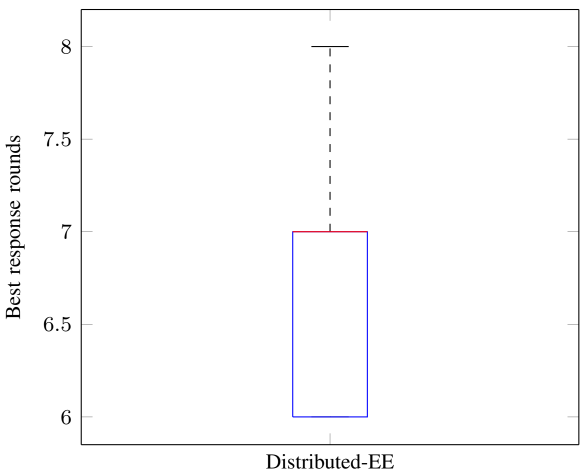

Fig. 10. Number of Best Response rounds of the distributed approach

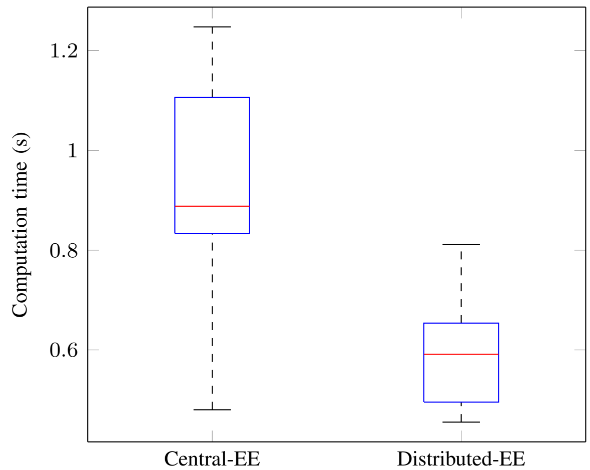

Fig. 11. Computation time of the centralized and distributed approaches

We display in Fig. 11 the total computation time[1] of the centralized and distributed approaches. Both approaches have very fast computation time with median sub-second values. This computation time is compatible with a real implementation of the algorithms: the scheduling decision is precomputed and implemented at the BS level, whereas the power allocation is rapidly computed following any change in the radio conditions. This important issue is further discussed in the following section.

X. Discussion on Implementation Feasibility

After assessing the performance of the centralized and distributed approaches in the previous sections, we outline hereafter some practical guidelines to implement the proposed algorithms.

A. Implementing the Per-Cell Scheduling

To compute the optimal solution of the per-cell scheduling problem, it suffices to know the number of RBs in the system. Such information is available at the BS level. The output consists in a percentage of time that is allocated to a user on each RB. Therefore, a simple but efficient way of implementing the solution is to extend the Round-Robin scheduler in a way to allocate equal time shares to the users in the cell on each RB.

[1]Computation time is evaluated on a 2.4 GHz Intel Core i7 computer with 8 GB of memory size.

B. Implementing the Centralized Power Control

To compute the optimal solution of the multi-cell power control problem, it suffices to know the channel power gain on all RBs across all users. Such information can be made available at a central unit level using proper signaling. This central unit can be either one of the BSs configured by the operator for a given geographical region or a Base Band Unit (BBU). The latter appears in two practical deployments: Coordinated Multi-Point (CoMP) and Cloud-RAN [52].

In order to compute the signaling burden of a centralized power control, let us consider that each BS sends to the central unit a vector containing the channel power gain of the serviced users on all RBs. The size of the vector is equal to the number of users multiplied by the number of RBs in a cell. The latter ranges between 6 and 100 for LTE FDD systems. Thus, in the worst case, each BS sends 100 channel power gain values per user to the central unit. Note that signaling is only required when the channel power gain changes on a given RB: this will occur at most once each TTI of 1 ms, when considering fast fading [53]. Let us suppose that channel power gain is encoded using 8-bit words, then the maximum signaling burden added by the centralized power control is 800 bits/ms or 800 kbps for each user in a cell. For instance, considering 10 users per cell we obtain a total of 8 Mbps in the worst case. The additional signaling load is relatively low and will be anyhow part of any inter-cell interference coordination protocol integrated in LTE-Advanced systems.

As shown in Section IX-D, the convergence time of the centralized power control algorithm does not exceed 1.2 s. Obviously, this time can be further reduced when the algorithm is implemented on a BS or BBU computation engine. Hence, the optimal solution can be reached before any change in the inputs.

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C. Implementing the Distributed Power Control

Our game theory-based approach is fully distributed as decisions made by any BS are completely decentralized in the sense that no information from other BSs is needed. At each iteration, any BS needs to know the channel power gain of its serviced users and the SINR on all RBs to solve the local optimization problem $\hat{\mathcal{P}}_3(\boldsymbol{\pi})$. Note that the distributed approach does not necessitate any explicit information on the power allocation of other cells, contrary to the centralized approach. Particularly, the SINR information is already available to the BSs through standard signaling of an LTE system. In fact, the SINR can be easily inferred through the CQI (Channel Quality Indicator) sent every TTI by scheduled users.

In each round of our Best Response algorithm, BSs take turns according to a predefined sequencing and perform power allocation. We assume that the SINR resulting from the power allocation of any BS will be available no later than one TTI. As shown in Section IX-D, the NE is almost attained after two rounds and the network can fairly follow an eventual change in the initial radio conditions.

XI. Conclusion

The preponderant energy consumption at the BS level has heightened the necessity to focus on energy efficient resource management in cellular networks. Therefore, we have put the stress in this article on energy efficient joint scheduling and power control in downlink *multi-cell* OFDMA networks, which lies in the scope of green radio communications. We have covered this challenging two-faceted problem according to two widely adopted approaches in radio resource management, namely, the centralized approach and the distributed approach. We have introduced algorithms that compute *optimal* solutions for each approach and proved mathematically their convergence.

On the one hand, the centralized approach makes use of the Dinkelbach method that converges in only three iterations. We have also showed that the optimal solution is obtained in less than one second. On the other hand, we have proved that the distributed approach converges, also in less than one second, to a pure NE using simple *Best Response* dynamics. The rapid convergence of both approaches paves the way for realistic deployments.

We have assessed the bearing of our approaches over the state-of-the-art algorithms using extensive numerical simulations. In a low interference scenario, the energy efficiency of both the centralized and distributed approaches is higher than the reference methods. Particularly, the centralized approach largely surpasses the one that maximizes the spectral efficiency or the legacy maximum power approaches. However, this predominance is reduced in a high interference scenario.

In future work, we focus on the investigation of joint scheduling and power control algorithms under radio condition fluctuations due to fading and mobility, in addition to the impact of partial or erroneous knowledge about the radio channel state. In particular, the focal point of our future work will be the stability assessment of the centralized optimal solution and the distributed Nash equilibrium.

Appendix A
Solution of the Per-Cell Scheduling Problem

Let us start by writing down the following optimization problem:

$$(\mathcal{P}_x(\boldsymbol{\theta})): \quad \max_{\boldsymbol{\theta}}\{\sum_{i\in\mathcal{I}(j),k\in\mathcal{K}} \log(\theta_{ik})\} \tag{32a}$$

$$\text{subject to} \quad \sum_{i\in\mathcal{I}(j),k\in\mathcal{K}} \theta_{ik} \leq \mathcal{I}(j)|, \tag{32b}$$

$$0 \leq \theta_{ik} \leq 1,\ \forall i \in \mathcal{I}(j), \forall k \in \mathcal{K}. \tag{32c}$$

Constraints (32b) are obtained by adding constraints (13b) of the per-cell scheduling problem $(\hat{\mathcal{P}}_{1j})$. Thus, $(\mathcal{P}_x)$ is a relaxation of $(\hat{\mathcal{P}}_{1j})$. In the relaxed problem, (32b) are binding and can be replaced by equality constraints. Moreover, the objective is equivalent to $\prod_{i\in\mathcal{I}(j),k\in\mathcal{K}} \theta_{ik}$. Therefore, $(\mathcal{P}_x)$ consists in maximizing the product of variables with constant sum. The maximum is known to be reached at equality, and we have at optimality:

$$\forall i \in \mathcal{I}(j), \forall k \in \mathcal{K}, \theta^*_{ik} = \frac{|\mathcal{I}(j)|}{|\mathcal{I}(j)| \times |\mathcal{K}|} = \frac{1}{|\mathcal{K}|}. \tag{33}$$

It is straightforward to show that θ^*_{ik} is also a feasible solution of the per-cell scheduling problem $(\hat{\mathcal{P}}_{1j})$. Knowing that if an optimal solution of the relaxed problem is feasible for the original problem, then it is optimal for the original problem, we deduce that θ^*_{ik} is an optimal solution of $(\hat{\mathcal{P}}_{1j})$ and conclude the proof.

Appendix B
Convexity of the Multi-Cell Power Control Problem

In the following, we demonstrate that problem $(\hat{\mathcal{P}}_2)$ in (12) can be transformed into a convex optimization via a change of variables. Let us start by performing a variable change $\tilde{\pi}_{jk} = \log(\pi_{jk})$ and defining $\tilde{N}_0 = \log(N_0)$ and $\tilde{G}_{ijk} = \log(G_{ijk})$. The objective function in (12a) can be written as:

$$\begin{aligned}
&\sum_{j\in\mathcal{J},i\in\mathcal{I}(j),k\in\mathcal{K}} \log(\frac{\exp(\tilde{\pi}_{jk}+\tilde{G}_{ijk})}{\exp(\tilde{N}_0)+\sum_{j'\neq j}\exp(\tilde{\pi}_{j'k}+\tilde{G}_{ij'k})}) \\
&\quad - \eta(\sum_{j\in\mathcal{J},k\in\mathcal{K}} p^1_j \exp(\tilde{\pi}_{jk}) + \sum_{j\in\mathcal{J}} p^0_j) \\
&= \sum_{j\in\mathcal{J},i\in\mathcal{I}(j),k\in\mathcal{K}} (\tilde{\pi}_{jk}+\tilde{G}_{ij'k}) \\
&\quad - \sum_{j\in\mathcal{J},i\in\mathcal{I}(j),k\in\mathcal{K}} \log(\exp(\tilde{N}_0)+\sum_{j'\neq j}\exp(\tilde{\pi}_{j'k}+\tilde{G}_{ij'k})) \\
&\quad - \eta(\sum_{j\in\mathcal{J},k\in\mathcal{K}} p^1_j \exp(\tilde{\pi}_{jk}) + \sum_{j\in\mathcal{J}} p^0_j)
\end{aligned}$$

The first term of the objective is a linear function in $\tilde{\pi}_{jk}$, thus concave (and convex). The second term contains log-sum-exp expressions which are convex. The opposite of the sum of convex functions is concave, thus the second term is concave. Similarly, the opposite of the sum of exponentials is concave, which completes the proof of the concavity of the objective function.

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Consequently, the total power constraints in (12b) can be written as:

$$\sum_{k \in \mathcal{K}} \exp(\tilde{\pi}_{jk}) \leq p_j^{max}$$
$$\log(\sum_{k \in \mathcal{K}} \exp(\tilde{\pi}_{jk})) - \log(p_j^{max}) \leq 0$$

This constraint is thus convex by virtue of the properties of the log-sum-exp functions. The constraints in (12c) can also be written as linear constraints:

$$\exp(\tilde{\pi}_{jk}) \geq p^{min}$$
$$-\tilde{\pi}_{jk} + \log(p^{min}) \leq 0$$

Knowing that the maximization of a concave objective function subject to convex constraints is a convex optimization problem, we conclude on the convexity of the transformed problem.

REFERENCES

[1] Y. Chen, S. Zhang, S. Xu, and G. Li, "Fundamental trade-offs on green wireless networks," *IEEE Communications Magazine*, vol. 49, no. 6, pp. 30–37, June 2011.

[2] A. D. Domenico, E. C. Strinati, and A. Capone, "Enabling green cellular networks: A survey and outlook," *Computer Communications*, vol. 37, pp. 5 – 24, 2014.

[3] A. Kwasinski and A. Kwasinski, "Increasing sustainability and resiliency of cellular network infrastructure by harvesting renewable energy," *IEEE Communications Magazine*, vol. 53, no. 4, pp. 110–116, April 2015.

[4] F. Richter, A. Fehske, and G. Fettweis, "Energy efficiency aspects of base station deployment strategies for cellular networks," in *Vehicular Technology Conference Fall (VTC 2009-Fall), 2009 IEEE 70th*, Sept 2009, pp. 1–5.

[5] J. Huang, V. Subramanian, R. Agrawal, and R. Berry, "Joint scheduling and resource allocation in uplink OFDM systems for broadband wireless access networks," *IEEE Journal on Selected Areas in Communications*, vol. 27, no. 2, pp. 226–234, February 2009.

[6] E. Yaacoub and Z. Dawy, "Joint uplink scheduling and interference mitigation in multicell LTE networks," in *IEEE International Conference on Communications (ICC)*, June 2011, pp. 1–5.

[7] Z. Lu, Y. Yang, X. Wen, Y. Ju, and W. Zheng, "A cross-layer resource allocation scheme for ICIC in LTE-advanced," *Journal of Network and Computer Applications*, vol. 34, no. 6, pp. 1861 – 1868, 2011.

[8] B. Maaz, K. Khawam, S. Tohme, S. Martin, S. Lahoud, and J. Nasreddine, "Joint scheduling and power control in multi-cell networks for inter-cell interference coordination," in *IEEE 11th International Conference on Wireless and Mobile Computing, Networking and Communications (WiMob)*, Oct 2015, pp. 778–785.

[9] J. Andrews, S. Buzzi, W. Choi, S. Hanly, A. Lozano, A. Soong, and J. Zhang, "What will 5g be?" *Selected Areas in Communications, IEEE Journal on*, vol. 32, no. 6, pp. 1065–1082, June 2014.

[10] L. Budzisz, F. Ganji, G. Rizzo, M. Marsan, M. Meo, Y. Zhang, G. Koutitas, L. Tassiulas, S. Lambert, B. Lannoo, M. Pickavet, A. Conte, I. Haratcherev, and A. Wolisz, "Dynamic resource provisioning for energy efficiency in wireless access networks: A survey and an outlook," *Communications Surveys Tutorials, IEEE*, vol. 16, no. 4, pp. 2259–2285, Fourth quarter 2014.

[11] J. Rao and A. Fapojuwo, "A survey of energy efficient resource management techniques for multicell cellular networks," *Communications Surveys Tutorials, IEEE*, vol. 16, no. 1, pp. 154–180, First quarter 2014.

[12] T. Yang, F. Heliot, and C. H. Foh, "A survey of green scheduling schemes for homogeneous and heterogeneous cellular networks," *Communications Magazine, IEEE*, vol. 53, no. 11, pp. 175–181, November 2015.

[13] W. Dinkelbach, "On nonlinear fractional programming," *Management Science*, vol. 13, no. 7, pp. 492–498, 1967.

[14] G. Miao, N. Himayat, and G. Li, "Energy-efficient link adaptation in frequency-selective channels," *Communications, IEEE Transactions on*, vol. 58, no. 2, pp. 545–554, February 2010.

[15] C. Xiong, G. Li, S. Zhang, Y. Chen, and S. Xu, "Energy-efficient resource allocation in OFDMA networks," *Communications, IEEE Transactions on*, vol. 60, no. 12, pp. 3767–3778, December 2012.

[16] Q. Shi, W. Xu, D. Li, Y. Wang, X. Gu, and W. Li, "On the energy efficiency optimality of OFDMA for SISO-OFDM downlink system," *Communications Letters, IEEE*, vol. 17, no. 3, pp. 541–544, March 2013.

[17] J. Tang, D. So, E. Alsusa, and K. Hamdi, "Resource efficiency: A new paradigm on energy efficiency and spectral efficiency tradeoff," *Wireless Communications, IEEE Transactions on*, vol. 13, no. 8, pp. 4656–4669, Aug 2014.

[18] Z. Ren, S. Chen, B. Hu, and W. Ma, "Energy-efficient resource allocation in downlink ofdm wireless systems with proportional rate constraints," *Vehicular Technology, IEEE Transactions on*, vol. 63, no. 5, pp. 2139–2150, Jun 2014.

[19] L. Xu, G. Yu, and Y. Jiang, "Energy-efficient resource allocation in single-cell ofdma systems: Multi-objective approach," *Wireless Communications, IEEE Transactions on*, vol. 14, no. 10, pp. 5848–5858, Oct 2015.

[20] Y. Li, M. Sheng, C. W. Tan, Y. Zhang, Y. Sun, X. Wang, Y. Shi, and J. Li, "Energy-efficient subcarrier assignment and power allocation in ofdma systems with max-min fairness guarantees," *Communications, IEEE Transactions on*, vol. 63, no. 9, pp. 3183–3195, Sept 2015.

[21] Q. Wu, W. Chen, M. Tao, J. Li, H. Tang, and J. Wu, "Resource allocation for joint transmitter and receiver energy efficiency maximization in downlink ofdma systems," *Communications, IEEE Transactions on*, vol. 63, no. 2, pp. 416–430, Feb 2015.

[22] G. Yu, Q. Chen, R. Yin, H. Zhang, and G. Ye Li, "Joint downlink and uplink resource allocation for energy-efficient carrier aggregation," *Wireless Communications, IEEE Transactions on*, vol. 14, no. 6, pp. 3207–3218, June 2015.

[23] L. Venturino, C. Risi, S. Buzzi, and A. Zappone, "Energy-efficient coordinated user scheduling and power control in downlink multi-cell OFDMA networks," in *Personal Indoor and Mobile Radio Communications (PIMRC), 2013 IEEE 24th International Symposium on*, Sept 2013, pp. 1655–1659.

[24] L. Venturino, A. Zappone, C. Risi, and S. Buzzi, "Energy-efficient scheduling and power allocation in downlink ofdma networks with base station coordination," *Wireless Communications, IEEE Transactions on*, vol. 14, no. 1, pp. 1–14, Jan 2015.

[25] A. Zappone, L. Sanguinetti, G. Bacci, E. Jorswieck, and M. Debbah, "Energy-efficient power control: A look at 5g wireless technologies," *Signal Processing, IEEE Transactions on*, vol. PP, no. 99, pp. 1–1, 2015.

[26] Z. Fei, C. Xing, N. Li, and J. Kuang, "Adaptive multiobjective optimisation for energy efficient interference coordination in multicell networks," *Communications, IET*, vol. 8, no. 8, pp. 1374–1383, May 2014.

[27] X. Sun and S. Wang, "Resource allocation scheme for energy saving in heterogeneous networks," *Wireless Communications, IEEE Transactions on*, vol. 14, no. 8, pp. 4407–4416, Aug 2015.

[28] D. Tsilimantos, J. Gorce, K. Jaffres-Runser, and H. Vincent Poor, "Spectral and energy efficiency trade-offs in cellular networks," *Wireless Communications, IEEE Transactions on*, vol. 15, no. 1, pp. 54–66, Jan 2016.

[29] G. Miao, N. Himayat, G. Li, A. Koc, and S. Talwar, "Interference-aware energy-efficient power optimization," in *Communications, 2009. ICC '09. IEEE International Conference on*, June 2009, pp. 1–5.

[30] G. Miao, N. Himayat, G. Li, and S. Talwar, "Distributed interference-aware energy-efficient power optimization," *Wireless Communications, IEEE Transactions on*, vol. 10, no. 4, pp. 1323–1333, April 2011.

[31] S. Bu and F. Yu, "Distributed energy-efficient resource allocation with fairness in wireless multicell ofdma networks," in *Global Communications Conference (GLOBECOM), 2014 IEEE*, Dec 2014, pp. 4708–4713.

[32] H.-H. Nguyen and W.-J. Hwang, "Distributed scheduling and discrete power control for energy efficiency in multi-cell networks," *Communications Letters, IEEE*, vol. 19, no. 12, pp. 2198–2201, Dec 2015.

[33] S. Buzzi, G. Colavolpe, D. Saturnino, and A. Zappone, "Potential games for energy-efficient power control and subcarrier allocation in uplink multicell ofdma systems," *Selected Topics in Signal Processing, IEEE Journal of*, vol. 6, no. 2, pp. 89–103, April 2012.

[34] F. Shams, G. Bacci, and M. Luise, "Energy-efficient power control for multiple-relay cooperative networks using q -learning," *Wireless Communications, IEEE Transactions on*, vol. 14, no. 3, pp. 1567–1580, March 2015.

[35] G. Bacci, E. Belmega, P. Mertikopoulos, and L. Sanguinetti, "Energy-aware competitive power allocation for heterogeneous networks under qos constraints," *Wireless Communications, IEEE Transactions on*, vol. 14, no. 9, pp. 4728–4742, Sept 2015.

[36] G. Lim, C. Xiong, L. Cimini, and G. Li, "Energy-efficient resource allocation for OFDMA-based multi-RAT networks," *Wireless Communications, IEEE Transactions on*, vol. 13, no. 5, pp. 2696–2705, May 2014.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSAC.2016.2611847, IEEE Journal on Selected Areas in Communications

[37] N. Z. Shor, K. C. Kiwiel, and A. Ruszcayǹski, *Minimization Methods for Non-differentiable Functions*. New York, NY, USA: Springer-Verlag New York, Inc., 1985.

[38] G. Auer, O. Blume, V. Giannini, I. Godor, M. Imran, Y. Jading, E. Katranaras, M. Olsson, D. Sabella, P. Skillermark *et al.*, "D2. 3: Energy efficiency analysis of the reference systems, areas of improvements and target breakdown," INFSOICT-247733 EARTH (Energy Aware Radio and NeTwork TecHnologies), Tech. Rep., 2010.

[39] A. Zappone and E. Jorswieck, "Energy efficiency in wireless networks via fractional programming theory," *Found. Trends Commun. Inf. Theory*, vol. 11, no. 3-4, pp. 185–396, Jun. 2015.

[40] F. Kelly, "Charging and rate control for elastic traffic," *European Transactions on Telecommunications*, vol. 8, no. 1, pp. 33–37, 1997.

[41] S. Boyd, L. Xiao, A. Mutapcic, and J. Mattingley, "Notes on decomposition methods," *Notes for EE364B, Stanford University*, pp. 1–36, 2007.

[42] D. Palomar, "Convex primal decomposition for multicarrier linear MIMO transceivers," *Signal Processing, IEEE Transactions on*, vol. 53, no. 12, pp. 4661–4674, Dec 2005.

[43] M. Chiang, "Balancing transport and physical layers in wireless multihop networks: jointly optimal congestion control and power control," *Selected Areas in Communications, IEEE Journal on*, vol. 23, no. 1, pp. 104–116, Jan 2005.

[44] J. B. Rosen, "Existence and uniqueness of equilibrium points for concave n-person games," *Econometrica*, vol. 33, no. 3, pp. 520–534, 1965.

[45] D. M. Topkis, "Equilibrium points in nonzero-sum n-person submodular games," *SIAM Journal on Control and Optimization*, vol. 17, no. 6, pp. 773–787, 1979.

[46] D. D. Yao, "S-modular games, with queueing applications," *Queueing Systems*, vol. 21, no. 3-4, pp. 449–475, 1995.

[47] L. S. Shapley, "A solution containing an arbitrary closed component," Ann. Math. Stud. No 40, 87-93, 1959.

[48] L. Hentilä, P. Kyösti, M. Käske, M. Narandzic, and M. Alatossava, "MATLAB implementation of the WINNER phase II channel model ver1.1," [Online]. Available: https://www.ist-winner.org/phase_2_model.html, December 2007.

[49] H. Claussen, "Efficient modelling of channel maps with correlated shadow fading in mobile radio systems," in *Personal, Indoor and Mobile Radio Communications, 2005. PIMRC 2005. IEEE 16th International Symposium on*, vol. 1, Sept 2005, pp. 512–516.

[50] T. Roughgarden, *Selfish Routing and the Price of Anarchy*. The MIT Press, 2005.

[51] P. J. Rousseeuw, I. Ruts, and J. W. Tukey, "The bagplot: A bivariate boxplot," *The American Statistician*, vol. 53, no. 4, pp. 382–387, 1999.

[52] D. Lee, H. Seo, B. Clerckx, E. Hardouin, D. Mazzarese, S. Nagata, and K. Sayana, "Coordinated multipoint transmission and reception in LTE-advanced: deployment scenarios and operational challenges," *Communications Magazine, IEEE*, vol. 50, no. 2, pp. 148–155, February 2012.

[53] B. Sklar, "Rayleigh fading channels in mobile digital communication systems .I. characterization," *Communications Magazine, IEEE*, vol. 35, no. 7, pp. 90–100, Jul 1997.

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This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JSAC.2016.2611847, IEEE Journal on Selected Areas in Communications

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